

Recent Advances in the Governing Form of Traffic for Bridge Loading

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ABSTRACT: The assessment of site-specific bridge traffic loading using WIM data is a critical feature of minimizing the cost of rehabilitation and replacement for bridge stock. For short- to medium-span bridges, it is often assumed that free-flowing traffic, including the dynamic interaction between the vehicles and bridge, governs the extreme load effect. In this paper, some recent advances in statistical techniques applied to bridge load effect extrapolation are presented. A critical review of these new approaches is made and it is shown that extrapolation results are now considerably more reliable and repeatable. It is also shown that there is doubt over the governing form of traffic. Therefore, the authors present some initial results of congested-traffic models in comparison to a free-flowing model. For a range of bridge lengths and load effects, the authors determine the dynamic ratio that would be required for free-flowing traffic to govern. The implications of these recent advances and various findings are discussed with reference to the future direction of research into bridge traffic loading

1 INTRODUCTION

1.1 Background

Ageing bridge stocks across the world mean that maintenance costs are an increasing proportion of road infrastructure expenditure. The EU expenditure on the repair, rehabilitation and maintenance of bridge structures is estimated to be €4-6 bn annually (COST 345). As only the 15 member states up to May 2004 are included in this estimate, in the 27-state EU, bridge maintenance expenditure is likely to be significantly more than €6 bn annually.

As a result of the increasingly high maintenance cost, research into the assessment of existing infrastructure has come into focus as significant savings are possible. Such savings can be made through more accurate modelling of both the physical and statistical phenomena associated with the problem. In particular, given that bridge traffic loading is significantly more variable than bridge capacity, it is in this area that much progress towards reducing maintenance expenditure may be made.

1.2 Basis of Research

Modelling bridge traffic load effect requires the input of actual highway traffic data, obtained from suitable installations. Weigh-In-Motion (WIM) technology is frequently used for this purpose. In this work five working days of WIM data was taken

from the A6 Paris to Lyon motorway near Auxerre, France. The site has 4 lanes of traffic (2 in each direction) but only the traffic recorded in the slow lanes was used (it is acknowledged that this results in conservative loading for a 2-lane bridge, for example). Truck traffic characteristics, such as weight and dimensional data, were collected for 17 756 and 18 617 trucks in the north and south slow lanes respectively (an average daily truck flow of 6744 trucks). The model used for the distribution of headways is particularly important as described by OBrien & Caprani (2005).

Monte Carlo simulation of traffic streams, based upon the measured traffic characteristics is performed (Caprani 2005). The bridge loading induced by such a traffic stream is then obtained by using influence lines (whether theoretical, site-measured, or obtained from finite-element modelling of the bridge) for the load effect of interest. We consider bridges with two opposing lanes of lengths in the range 20 to 50 m. The load effects examined are:

- Load Effect 1: Bending moment at the mid-span of a simply-supported bridge;
- Load Effect 2: Bending moment at the central support of a two-span continuous bridge;
- Load Effect 3: Left hand shear in a simply-supported bridge.

Only significant crossing events, defined as multiple-truck presence events and single truck events where the vehicle's Gross Vehicle Weight (GVW) is

in excess of 40 tonnes, were processed to minimize computing requirements. For such events, the comprising truck(s) are moved in 0.02 second intervals across the bridge and the maximum load effects of interest identified. The set of daily maximum load effect values for each loading event type were determined for further statistical analysis.

IN this work traffic growth is not considered and consequently the statistical models applied are stationary. In addition, it is taken that the 'economic year' is equivalent to about 50 weeks of weekday traffic and consequently 250 'simulation days' are taken to represent a calendar year.

2 STATIC TRAFFIC LOAD EFFECT

2.1 *Statistical Methods in the Literature*

2.1.1 *Attributes of Good Statistical Modelling*

The attributes required of a robust statistical extrapolation procedure are described by Caprani (2005) and summarized here. The most important property is that a model should not be subjective: different results obtained as a result of the analyst's decisions should be avoided. Other requirements are:

- Choice of Population: The population upon which the analysis is based must be in keeping with the limitations of the statistical model to be applied. In many cases the stationarity assumption of many statistical models is violated.
- Distribution of Extreme Load Effects: Often decisions about which extreme value distribution to use are made in block maxima analyses. This is unnecessary given that the Generalized Extreme Value (GEV) distribution (Coles 2001) incorporates all three Fisher-Tippett families.
- Estimation: The means by which the model parameter estimation is done is often graphical or least-squares-based when more accurate methods, such as maximum likelihood estimation exist.
- Choice of Thresholds: Many authors make decisions regarding the data which is to be kept as a basis for the analysis – the 'tail' data problem. This is unnecessary if the correct model is being applied to the correct population using good estimation procedures.

2.1.2 *Statistical Methods in the Literature*

In the bridge traffic load effect literature, load effects have been found from various methods, but it is the methods of extrapolating this load effect data that is of interest here.

In the background studies for the development of the Eurocode for bridge loading (EC1.2 2003), Bruls et al. (1996) and Flint & Jacob (1996) consider various methods of extrapolation, including:

- a half-normal curve fitted to the histogram tail;
- a Gumbel distribution fit to the histogram tail;

- Rice's formula for a stationary Gaussian process;

Rice's formula has been used extensively in the literature (Flint & Jacob 1996, Cremona 2001). This method involves the choice of a threshold; Cremona (2001) develops an optimal level at which to set the threshold, based on minimization of the Kolmogorov-Smirnov statistic.

In the papers Nowak (1989) and Nowak & Hong (1991), straight lines are fit to the tails of the load effect distributions plotted on normal probability paper. Nowak (1994) uses curved lines to extrapolate for the load effects of various return periods whilst Nowak (1993) determines the distribution of maximum load effect by raising the parent distribution of load effect to an appropriate power. In this way he determines the mean and coefficient of variation of the maximum load effect. Fu & Hag-Elsafi (1995) also obtain the distribution of maximum load effect by raising the parent distribution to an appropriate power. Similarly, Ghosn & Moses (1985) use a 2.4 hour maximum as their extreme data which is then fitted and raised to the appropriate power to obtain the 50-year load effect distribution. Cooper (1995, 1997) also raises the distribution of measured load effect to a power to get the 4.5-day distribution of maximum load effect. This is modelled with a Gumbel distribution, which is used to extrapolate to a 2400-year return period.

Buckland et al. (1980) use a Gumbel distribution to fit 3-monthly maximum load effect which is then used to extrapolate to the return periods of interest. Bailey & Bez (1994 and 1999) determine that the Weibull distribution is most appropriate to model load effect tails and used maximum likelihood estimation. In Moyo et al. (2002), daily maximum bridge strain measurements are fit to a Gumbel distribution using least-squares on probability paper.

Lastly, but notably, Crespo-Minguillón & Casas (1997) adopt the peaks-over-threshold approach and use the Generalized Pareto Distribution to model the exceedances of weekly maximum traffic load effect over a threshold. An optimal threshold is selected based on the overall minimum least-squares value, and the distribution corresponding to this threshold is used as the basis for extrapolation.

2.1.3 *Conclusions*

Save for the approach of Crespo-Minguillón & Casas (1997), other means of extrapolation generally fail to meet one or more of the minimum requirements of a good statistical model. Further, as can be seen, variability of the characteristic load effect is not typically assessed. Extrapolations are carried out to the return period, rather than to find the actual characteristic value, which for the Eurocode (EC1.2 2003) is usually taken as 10% probability of exceedance in 100 years.

2.2 Recent Advances

2.2.1 The Nature of Bridge Traffic Load Effect

Recent work (Caprani et al. 2008) has concluded that bridge traffic load effect is not a single statistical generating mechanism. As is intuitively reasonable, the distribution of load effects caused by a 2-truck event (two trucks concurrently present on the bridge) differs to that of a 3-truck event. When each loading event-type is isolated, it is found that the GEV distribution is appropriate to model the daily maximum load effects that result (Caprani 2005). Thus a composite distribution of daily maximum load effect is required as a basis for extrapolation. Caprani et al. (2008) show that an appropriate model is the composite distribution statistics (CDS) model, $G_C(\cdot)$:

$$G_C(s) = \prod_{i=1}^N G_i(s) \quad (1)$$

where $G_i(\cdot)$ is any extreme value distribution. When the block maxima method is used (Coles 2001), the GEV distribution for loading event type i is:

$$G_i(s) = \exp \left\{ - \left[1 - \xi_i \left(\frac{s - \mu_i}{\sigma_i} \right) \right]_+^{1/\xi_i} \right\} \quad (2)$$

where $[h]_+ = \max(h, 0)$ and the parameters, μ_i , σ_i , ξ_i , are found by fitting to the load effect data of loading event type i solely. This model has been shown to exhibit greater fidelity in fitting distributions of load effect, and meets minimum requirements for a good extrapolation model (Caprani 2005).

2.2.2 Predicting the Lifetime Load Effect

Extrapolations to a return period result in a single value of load effect. Since repeating the process would generally yield a different result, there should be a means of acknowledging both this variability and the variability that arises from the modelling process itself. Since many codes define characteristic values as a probability of exceedance in the design life of the structure (for example, the Eurocode's 10% probability of exceedance in 100 years definition), it is not a distribution of characteristic values that is of interest, but the distribution of lifetime load effect. Therefore focus should be centred on the estimation of the lifetime distribution of load effect, from which the characteristic value can then be derived. Of significant further value would be a means by which allowances for modelling uncertainties, such as parameter confidence intervals, could be included.

Predictive likelihood is a method for estimation which allows both for sampling and modelling uncertainties. It is based on the maximization of the likelihood of both the data and a predictand (possible prediction value):

$$L_p(z|y) = \sup_{\theta} L_y(\theta; y) L_z(\theta; z) \quad (3)$$

where $L_p(z|y)$ is the predictive (joint) likelihood of the predictand z , given the data vector, y ; $L_y(\theta; y)$ is the likelihood of the parameter vector θ given the data y , and; $L_z(\theta; z)$ is the likelihood of the parameter vector θ given the predictand z . Since the likelihoods are jointly maximized, L_p gives an indication of the relative likelihood of the data giving rise to the predictand. Application of Equation (3) for a range of predictands allows a probability density function of predictands to be determined. See Caprani (2005) for a more detailed explanation.

Caprani & OBrien (2009) have applied this method to the bridge loading problem and showed that the traditional return period approach yields different results to the direct estimate of the characteristic value from the lifetime distribution of load effect (Caprani & OBrien 2006a). The method has also been shown Caprani & OBrien (2006b) to be effective in predicting extreme vehicle weights.

2.3 Comparison

The net effect of the application of the two advances just described, in comparison to a statistical model which represents the best of the models in the literature, is shown in Figure 1 for the three load effects being considered in this work.

From Figure 1 it can be seen that the differences are generally small, with notable exceptions for spans of about 40 m. In particular, Load Effect 2 is sensitive to spans around 40 m due to the shape of its influence line, and the inter-vehicle gaps. That the differences are not excessive, despite the advances in analysis, shows a certain degree of robustness amongst the better statistical extrapolation methods.

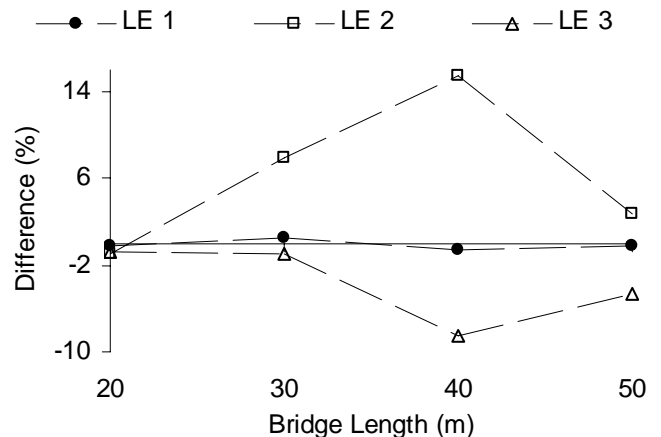


Figure 1. Change in predicted load effect due to recent advances.

3 TRAFFIC LOAD EFFECT ALLOWING FOR DYNAMIC INTERACTION

3.1 Allowing for Dynamic Interaction

The dynamic amplification factor (DAF) is defined as the ratio of total to static load effect, where total load effect results from the truck and bridge interacting dynamically. Allowances for dynamic interaction are made in bridge loading codes, based on the notion of the DAF. Usually however, the worst possible DAF is applied to the critical static load effect and this approach does not take into account the reduced likelihood of these events coinciding. Indeed it is intuitively reasonable that grossly overloaded vehicles are not as dynamically lively as unloaded vehicles, for example. Furthermore, it is also reasonable that critical static loading events, involving many vehicles, will have destructive interference of the dynamic behaviour, resulting in lower levels of dynamic interaction, on the average.

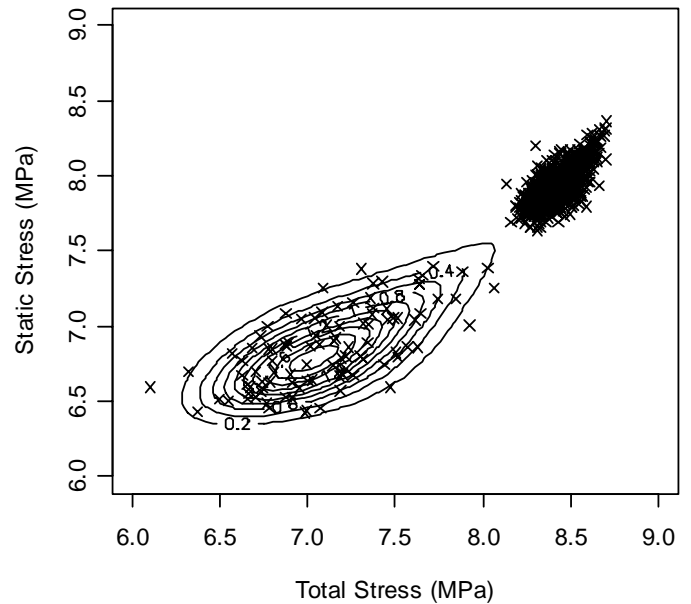
3.2 Dynamic Interaction at the Lifetime Load Effect

3.2.1 Statistical Background

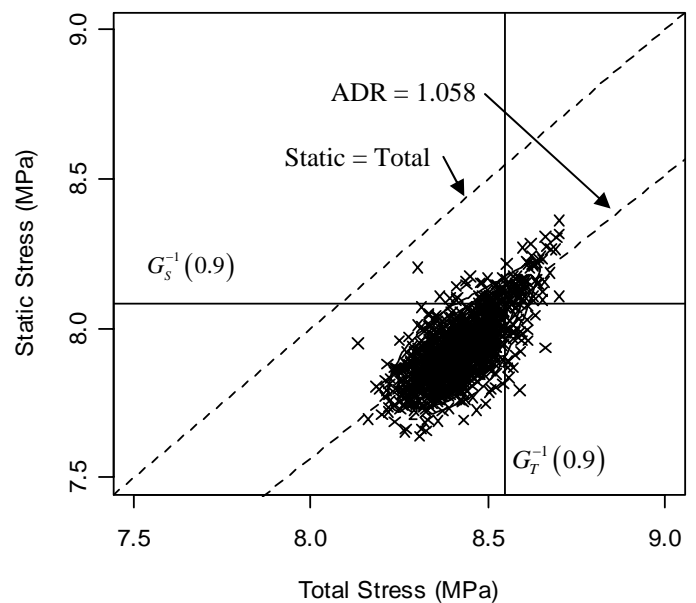
Total and static load effects are related through the DAF, which is not constant as all loading events differ both dynamically and statically. However, there remains a degree of correlation between these statistical variables. The recent statistical theory of multivariate extreme values has been applied to this problem to extrapolate these correlated variables to their design lifetime values (Caprani 2005). Their ratio at this level is therefore the level of dynamic interaction applicable for the bridge design lifetime. This has been termed the assessment dynamic ratio (ADR) in recognition that it does not arise from any one single loading event.

3.3 Sample Application

The Mura River bridge in Slovenia is used to provide a sample application of the statistical analysis for ADR. Monthly maximum mid-span bending stresses were identified from static simulations. These events then modelled to determine the level of dynamic interaction, as explained in Gonzalez et al (2008). The population of total and static load effects were analysed using a Gumbel Bivariate Extreme Value Distribution (BEVD). Parametric bootstrapping was then used to determine the lifetime BEVD, from which the relationship between characteristic total and characteristic static load effects was determined, the ratio of which is defined as the ADR, shown in Figure 2 (Caprani 2005). As can be seen, the expected level of lifetime dynamic interaction, for this site and bridge, is a DAF of about 1.06. This is significantly less than the DAF allowed for in the Eurocode of about 1.13 for such a bridge and load effect.



(a) Bivariate parent and extreme populations;



(b) Lifetime total and static load effect;

Figure 2. Bivariate extreme value extrapolation for ADR.

3.4 Implications for the General Bridge Traffic Load Effect Problem

The findings, just outlined, have significant implications for the assessment of lifetime bridge traffic load effect, as well as the direction that future research into the area should take. The ADR finding has particular importance given that the majority of bridges are of short- to medium-length since it is currently assumed that the governing loading scenario for these bridges is that of free-flowing traffic with associated dynamic effects. The low level of lifetime dynamic allowance found for the Mura River bridge, if found to be general, will alter the governing loading scenario for the vast majority of bridges as summarised in Figure 3.

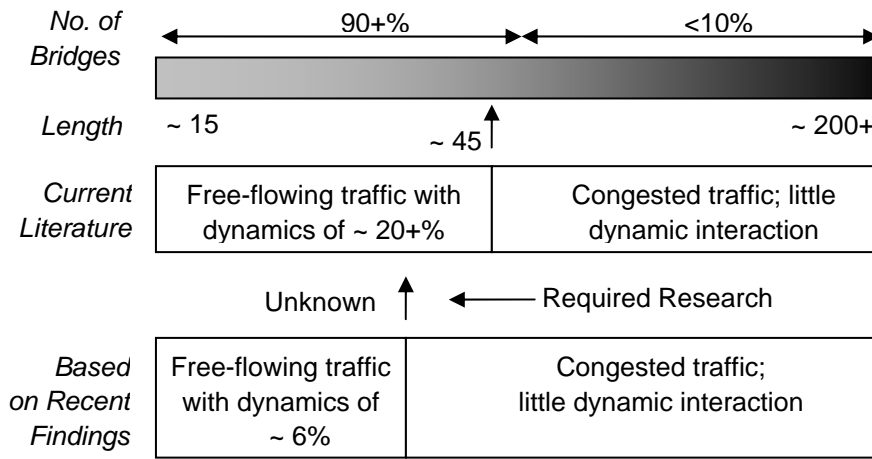


Figure 3. Governing loading scenarios for different bridge lengths and traffic load effect source.

4 THE GOVERNING FORM OF TRAFFIC

4.1 Traffic Modelling

To examine the issue of the governing form of traffic, several forms of traffic models are considered. For the free-flow models, the measured hourly flow rate of trucks of the Auxerre site are maintained, thereby eliminating truck volume as a variable. Also, the site-measured truck composition is used. A range of car percentages is also considered.

4.1.1 Standard Free-Flow Model (SFFM)

Free-flow traffic models have been used for many years to model highway bridge loading (Caprani 2005). Measured parameters such as speed and hourly flow rates can be maintained throughout a simulation. However, there remains the problem area of headway, or distance (in time) from the front of one truck to the front of the subsequent truck. O'Brien and Caprani (2005) propose a model that is sympathetic to the measured headway data, accounts for flow, and does not require subjective assessments of minimum gap and this model is used here with measured site flow properties to constitute a standard free-flow model (SFFM).

4.1.2 Standard Congestion Model (SCM)

Congested traffic modelling for loading on short- to medium-length bridges has not been studied extensively. Nowak and Hong (1991) modelled static configurations of traffic with assumed gaps of 15 ft (4.57 m) and 30 ft (9.14 m). Vrouwenvelder and Waarts (1993) use two models: for distributed lane loads a gap of 5.5 m is used, whilst for full modelling a variable gap of 4 to 10 m is used. In the background studies to the Eurocode (EC1.2 (2003)), Bruls et al (1996) and Flint and Jacob (1996) use a 5 m gap between vehicles.

In this study, the gap between vehicles is considered as a stochastic variable. Thus the standard congestion model (SCM) is taken to have the 5 m mean and a coefficient of variation of 5%, due to its prevalence in the literature.

4.1.3 Traffic Microsimulation Model (MS-IDM)

Traffic microsimulation is an ideal tool to counter many of the problems associated with previous traffic modelling for bridge loading. Microsimulation models the actual driving behaviour of vehicles on the roadway. For this work, the Intelligent Driver Model (IDM) developed by M. Treiber and others (Treiber et al 2000a, Treiber et al 2000b) is used as the microsimulation model. The IDM has a limited number of parameters and an intuitive algorithm. These authors have calibrated the IDM against data obtained for three German highways (Treiber et al 2000b). The IDM parameters used in this study are similar to these values, but are taken to be stochastic variables with small variation. Two relevant parameters are given the values:

- Desired velocity: taken as normally distributed; $N(110 \text{ km/h}, 7.0 \text{ km/h})$ for cars and $N(90 \text{ km/h}, 3.6 \text{ km/h})$ for trucks;
- Safe time headway: taken as normally distributed; $N(1.2 \text{ s}, 0.05 \text{ s})$ for cars and $N(1.5 \text{ s}, 0.05 \text{ s})$ for trucks;

A 2 km road section was used to simulate the vehicles movement using the IDM. A speed limit of 50 km/h was defined from 500 m to 1500 m. The arrival times of the vehicles at a virtual loop detector (located at the start of the speed limit region) were output, to give the headway between successive vehicles.

4.2 Application to Bridge Traffic Loading

Traffic streams were generated using both the SFFM and the SCM for a range of car percentages. Using influence lines and the statistical extrapolations ex-

plained previously, the lifetime load effects for a range of spans and load effects were found for these traffic scenarios. Traffic microsimulation of the SFFM and SCM traffic files was then carried out. These new traffic files were again processed for lifetime load effects.

Figure 4 shows the mean change in load effect that occurs by the application of traffic microsimulation. Immediately apparent is that load effect increases significantly when microsimulation is applied to the free-flow model when the traffic is comprised of 90% cars. This is explained by the large number of vehicles which result in congestion on the microsimulation road, even though the arrival times to the start of the road were generated according to a free-flow model. The final values of load effect are close to those caused by congested models, as would be expected. Also obvious is the general trend for load effects to reduce for all other traffic models and composition. In fact applying microsimulation to the congestion model reduces load effect significantly suggesting that the congested

models used for bridge load effect estimation are quite conservative. In contrast, except for the increase in load effect for 90% cars, the application of microsimulation to free-flow model-generated traffic results in smaller reduction in load effect. This is as may be expected since a free-flow model should closely resemble driving traffic.

4.3 The Governing Form of Traffic

To determine for what load effects and bridge lengths the different traffic regimes govern, it is useful to consider the value of DAF (or equivalently ADR) which is required in order for free-flowing traffic regimes to govern (Figure 5). Thus, as knowledge about lifetime DAF values becomes more available, it is easier to assess the governing form of traffic. As a simplification, we take the average load effect predictions from the three traffic compositions considered. Dividing the congested model results by the free-flow model results gives us this 'Required DAF'.

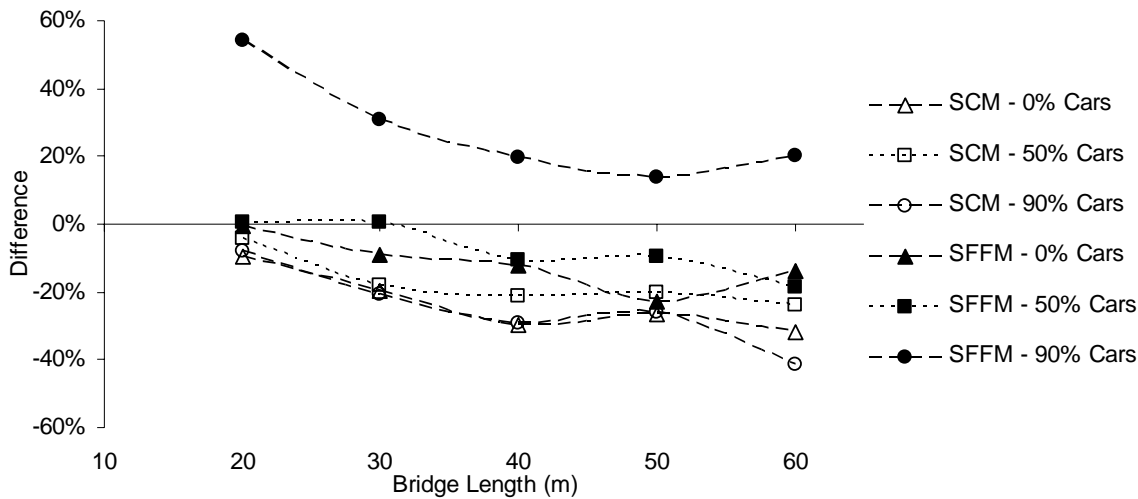


Figure 4. Impact of traffic microsimulation on load effects from standard free-flow and congested traffic models.

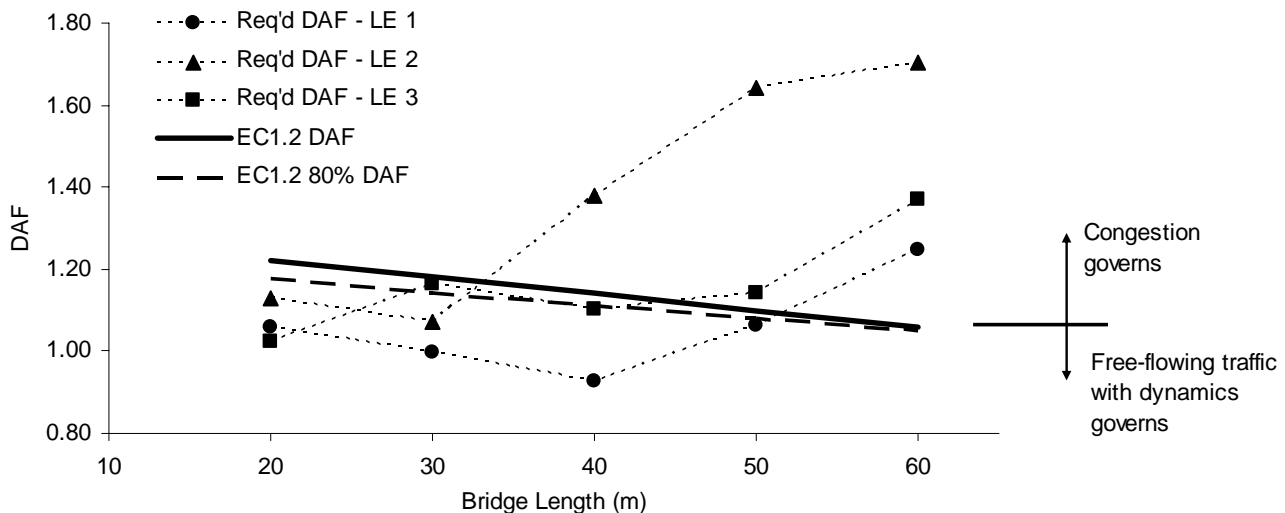


Figure 5. Identification of governing traffic state through required DAF.

Figure 5 shows the values of Required DAF for each load effect, along with the Eurocode values DAF for comparison. In this figure, once the required DAF is larger than the design DAF, congested traffic governs. Thus, from Figure 5, congested traffic governs above lengths of about 52 m, 33 m and 45 m, for Load Effects 1, 2 and 3 respectively.

It is also possible to assess the impact of a postulated reduction in the dynamic increment of 20%, as shown in Figure 5. For example, the DAF of 1.20 has an increment of 20% which, when reduced by 20% results in a DAF of 1.16 – called EC1.2 80% DAF in the figure. Depending on the slopes of the various lines, this change may have small or significant impact. Applying this 20% reduction in DAF, results in congestion governing for bridge lengths of about 50 m, 32 m and 38 m, for Load Effects 1, 2 and 3 respectively. Thus the governing traffic loading scenario for Load Effect 2 is sensitive to the value of DAF used.

5 SUMMARY & CONCLUSIONS

5.1 Summary

In this paper a number of advances in the statistical analysis of bridge traffic loading have been presented. A model that better represents the physical phenomenon has been presented as has a means of establishing the distribution of lifetime load effect, allowing for numerous sources of variability. Also, of significance, is a bivariate extreme value model that yields the dynamic amplification factor at the lifetime level. The cumulative effect of these findings is to challenge the idea that congested traffic only governs for spans over about 45 m.

To investigate the governing form of traffic loading, standard free-flow and congested traffic models were used to generate traffic streams which were then processed using traffic microsimulation. This has shown that congested traffic models are quite conservative. Further, the idea of a required DAF was introduced to show that different forms of traffic govern at different bridge lengths for different load effects. It was also shown that the length at which congestion begins to govern is sensitive to the codified DAF requirement.

5.2 Conclusions

A clear need for further research is evident from the advances presented here. For computing the dynamic interactions, advances are required so that traffic simulations could incorporate dynamics as the simulations progress, instead of requiring time-consuming post-processing outside the simulation. Also, the statistical methods presented need to be further advanced. For example, a multivariate peaks-

over-threshold approach would avoid the need for decisions as to block and population size. Indeed, if dynamic interaction is subsequently found to play only a small part in bridge lifetime loading, reductions in loading are more likely to come from advancing the statistical analyses applied to the problem. It was also found that traffic microsimulation, while better modelling real traffic, also yields lower traffic load effects in general. The proper calibration and extensive use of this technique could thus yield significant reductions in lifetime load effect.

Due to the critical importance of highway infrastructure and the consequent need for conservatism, many national bridge authorities are reluctant to allow bridge assessment consultants to operate outside codes of practice. The significant benefits that the ongoing research into the bridge loading problem can bring must therefore be brought to the attention of bridge owners and consultants. Concurrently, codes of practice must be updated, and where possible, provision made for the possibility of using proven state-of-the-art methods. It is only through such measures that the ultimate goal of bridge traffic load estimation will be realized.

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