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Dublin Institute of Technology & University College Dublin

ICOSSAR '09

13 September – 17 September 2009,

Osaka, Japan

Estimating Extreme Highway Bridge Traffic Load Effects

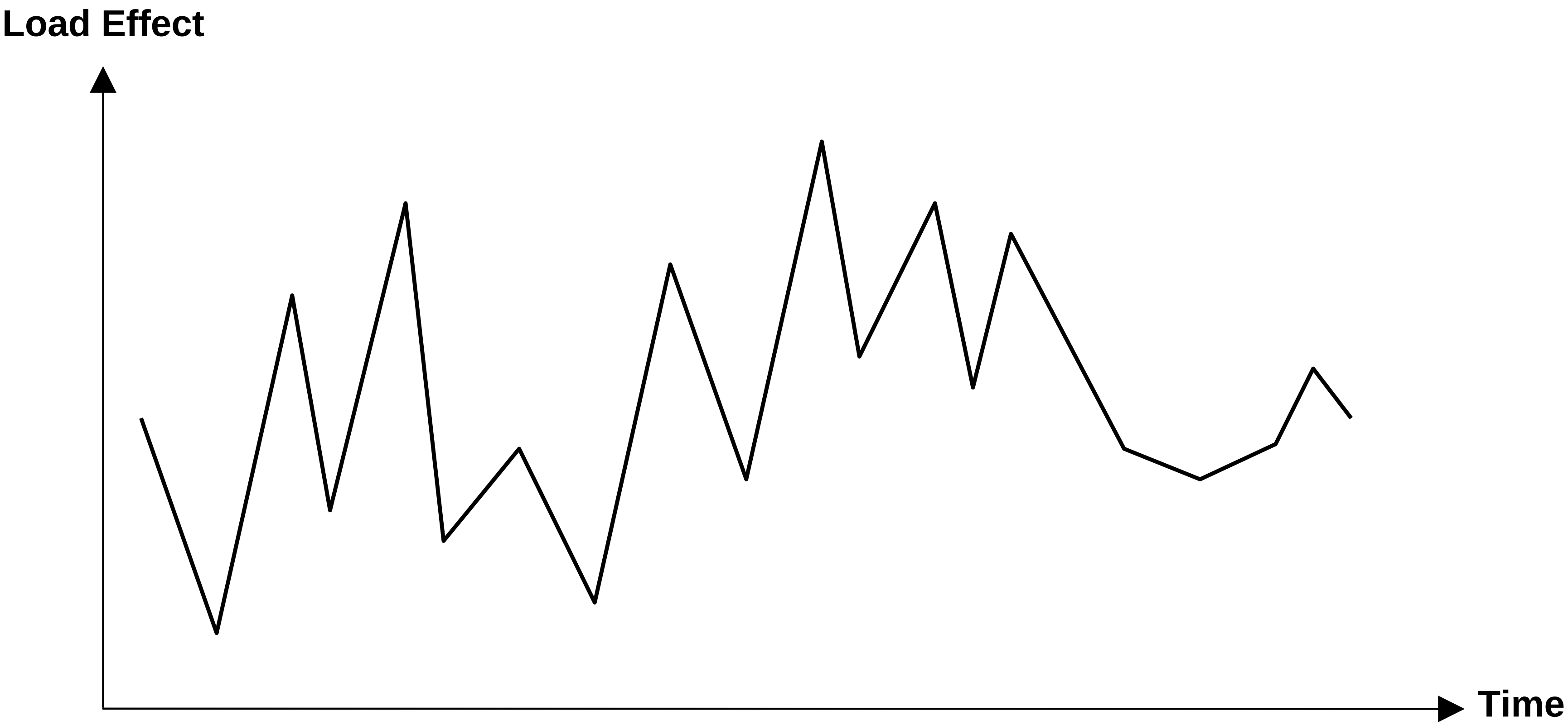
Introduction

Statistics in the hands of an engineer are like a lamppost to a drunk – they're used more for support than illumination.

- A. E. Housman.

Extreme Value Methods I

Block maxima approach – data modelled using GEV distribution:



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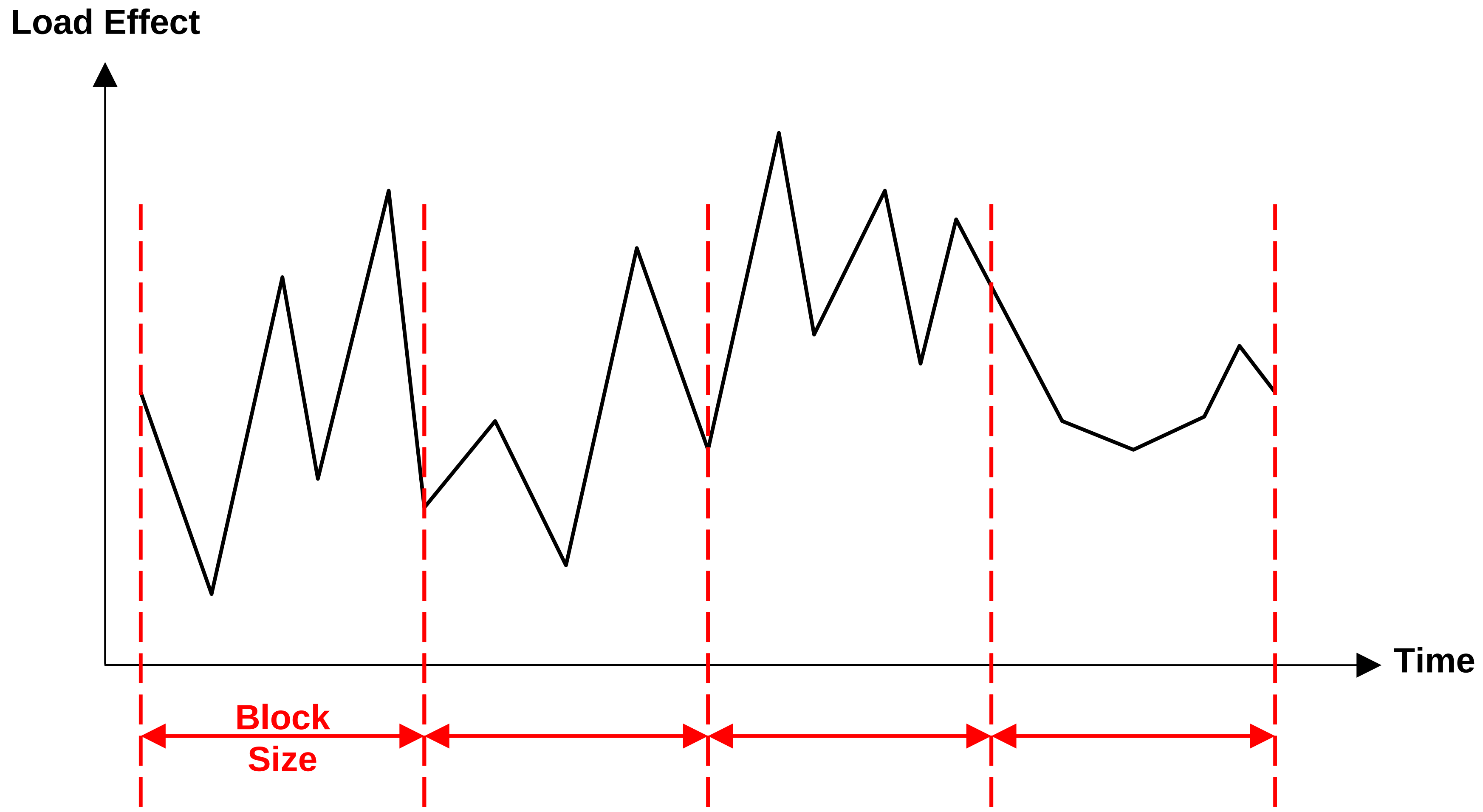

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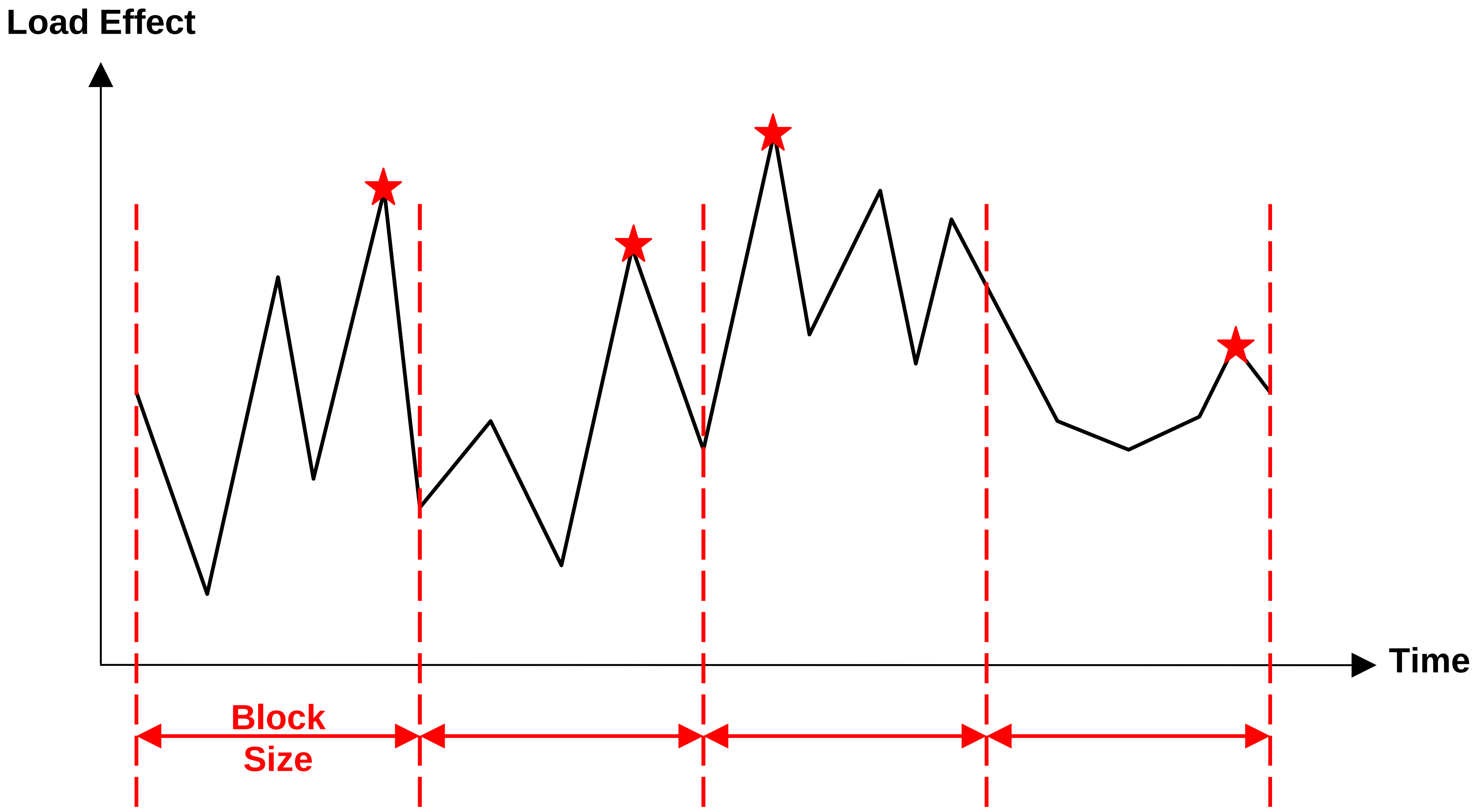
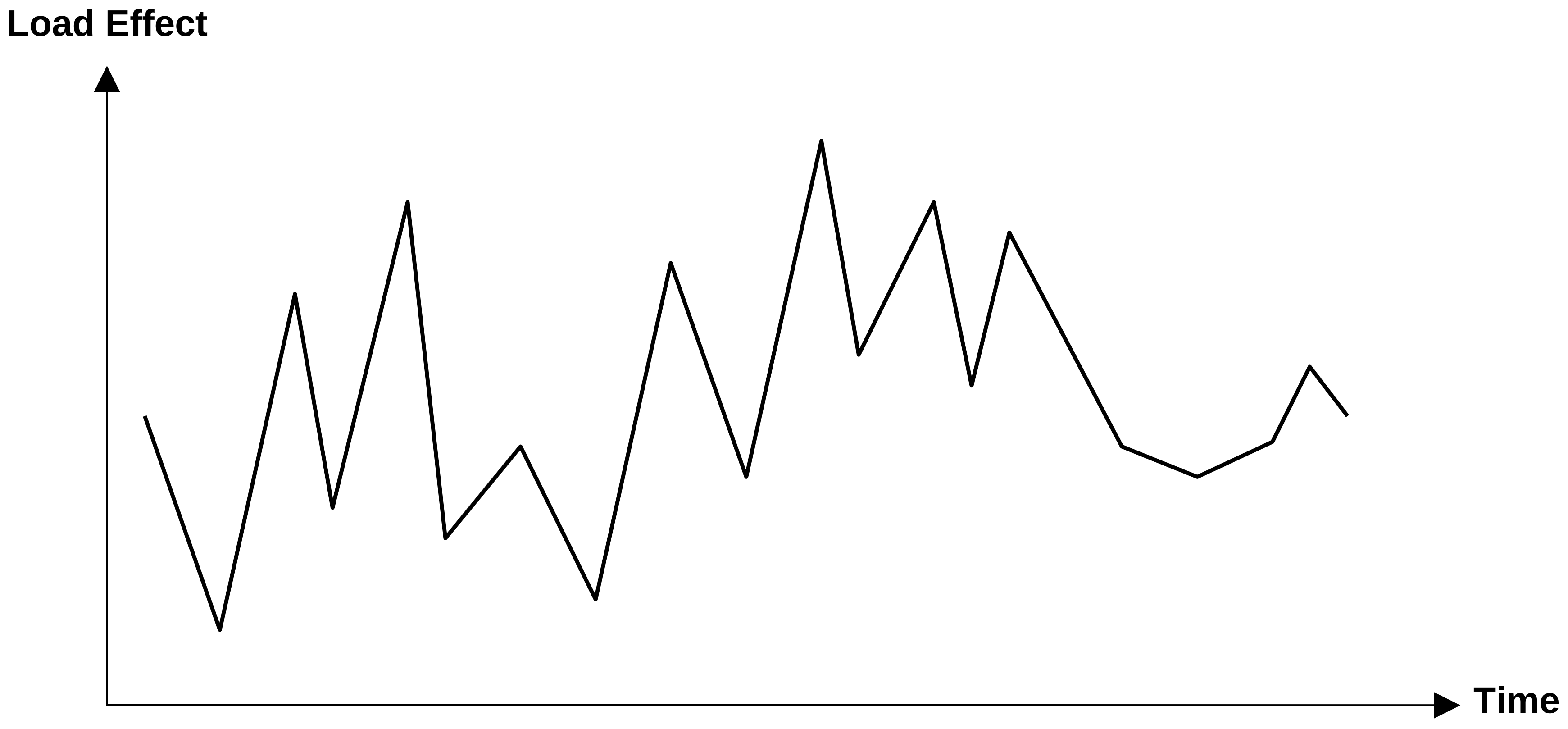


Extreme Value Methods II

Peaks Over Threshold (POT) approach – data modelled using GPD distribution:



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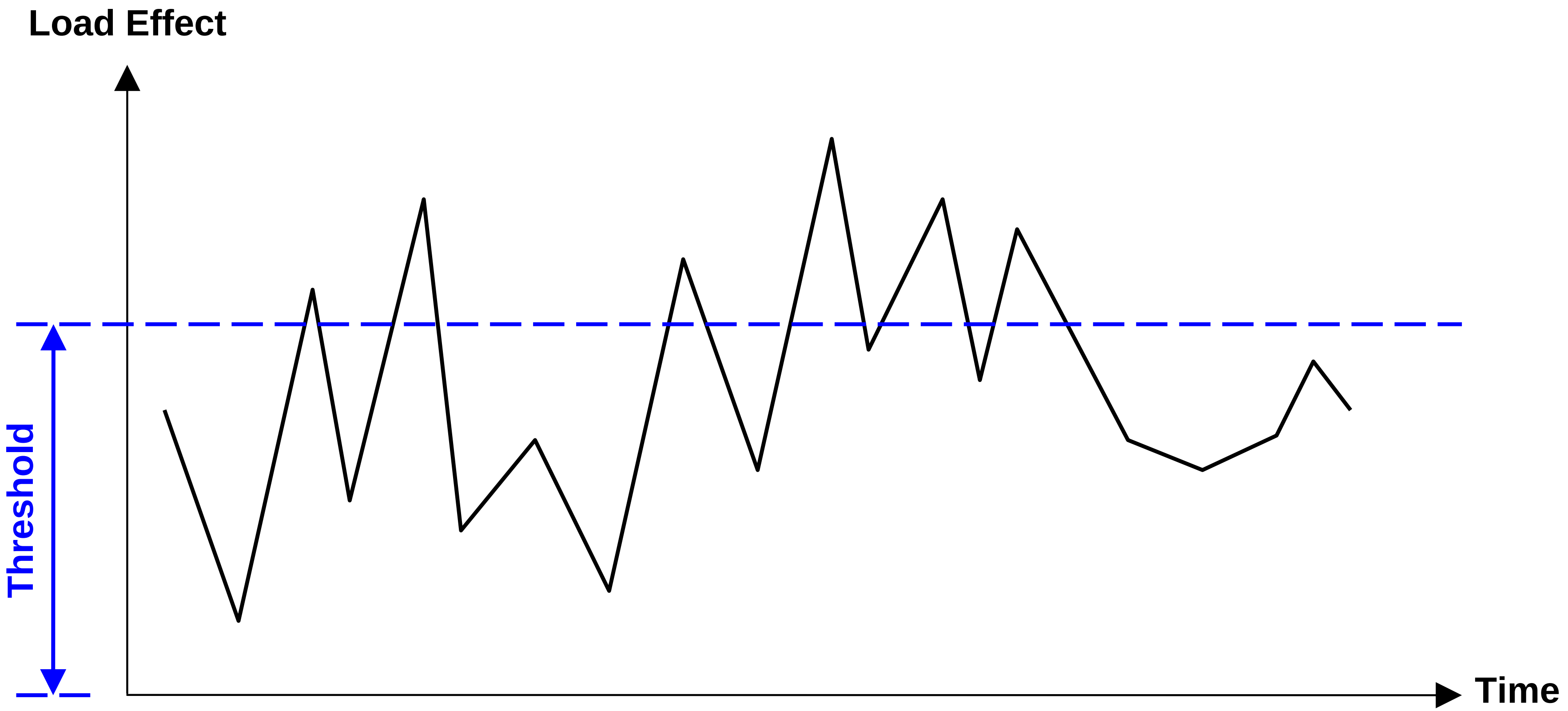

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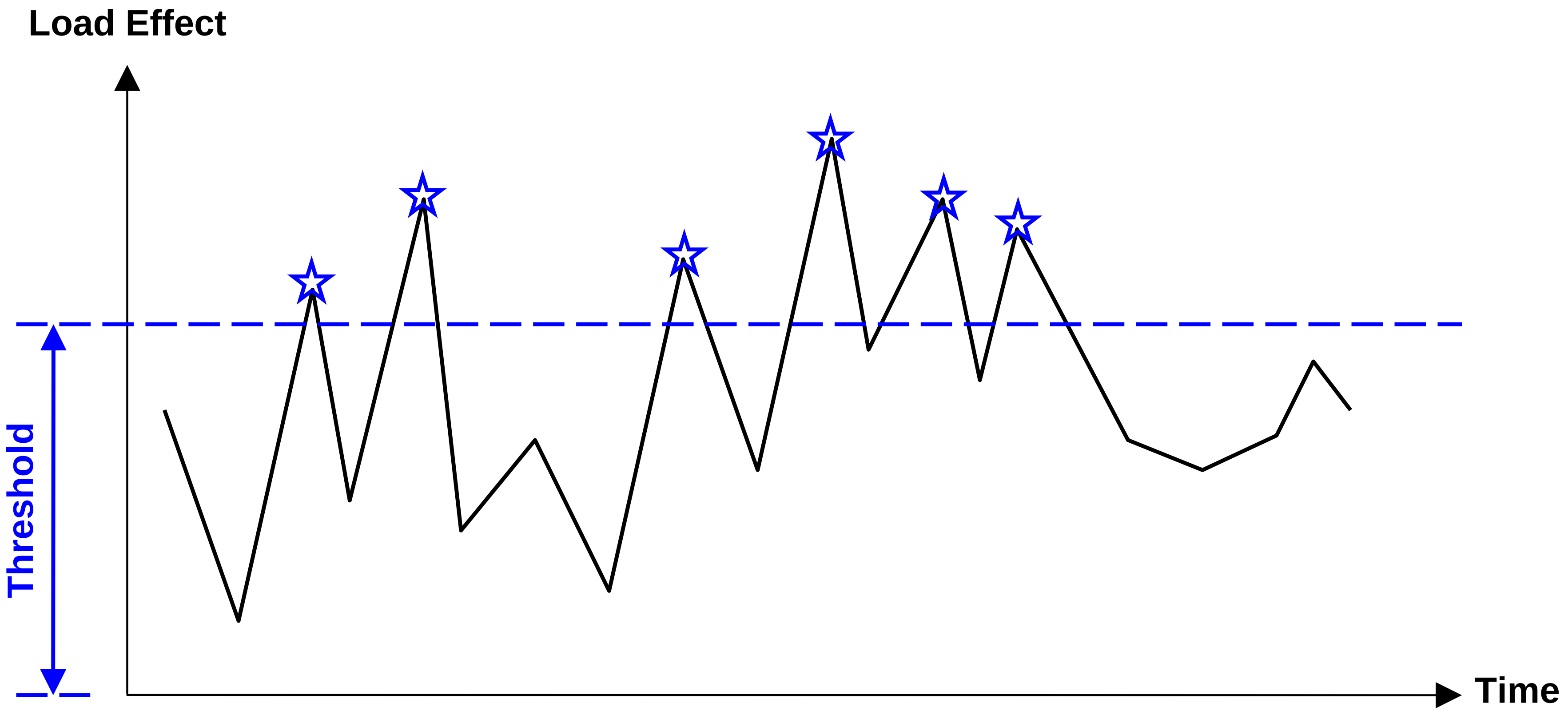

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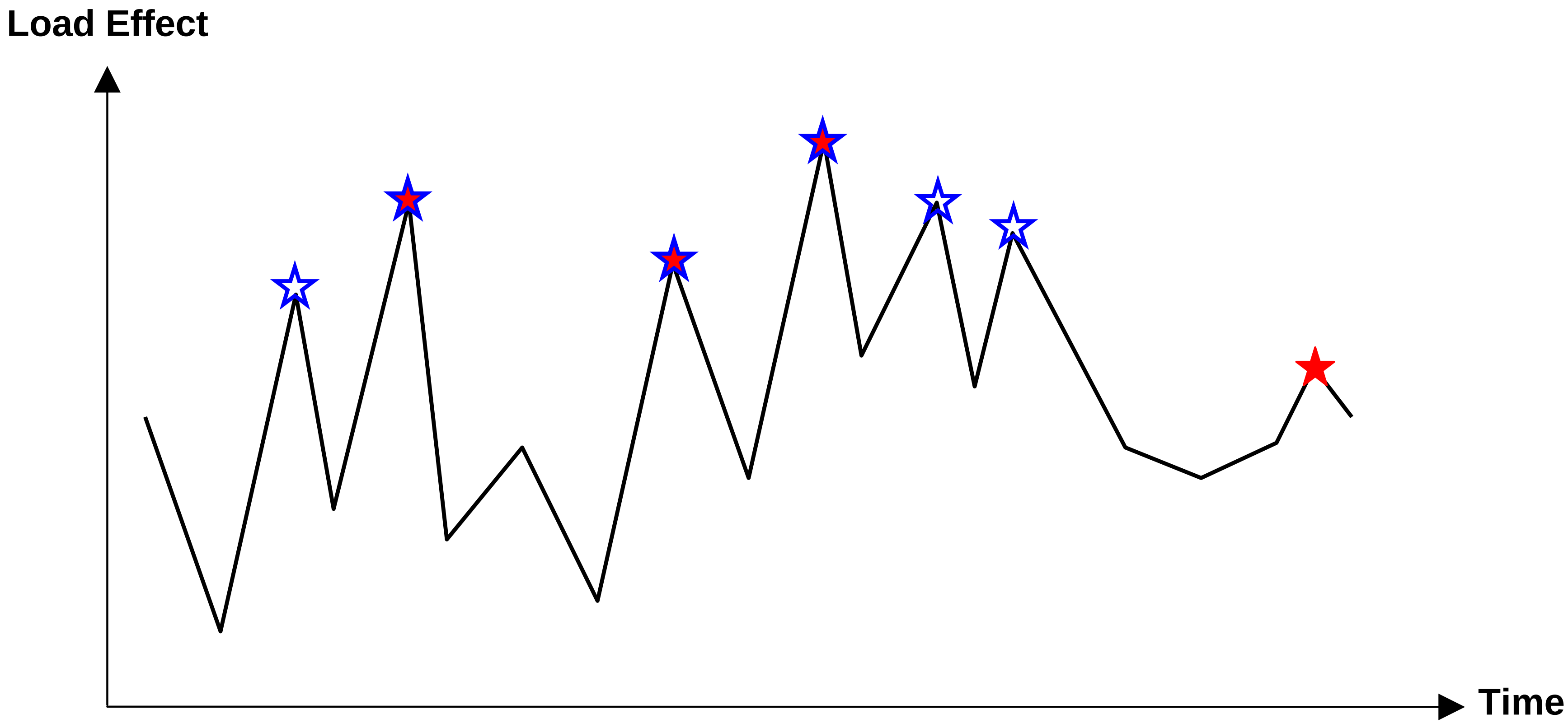
Extreme Value Methods III

Differences in the approaches:



Extreme Value Methods IV

Choice between the approaches:

- **Gives different results**
- **Often subjective**
- **Difficult to assess best model**

As the GEV and GPD are different distributions:

- **They are non-nested models**
- **Cannot calculate a statistical significance of differences**

Box-Cox GEV Distribution I

The BCGEV distribution:

- Introduced by Bali (2003) for use in economic modelling
- Includes both GEV and GPD distributions through a model parameter, λ
- Maintains the usual GEV/GPD parameter set, (μ, σ, ξ):

$$H(s) = \left(\frac{1}{\lambda}\right)\left(\left[\exp\left\{-\left[h(s)\right]_+^{1/\xi}\right\}\right]^{\lambda} - 1\right) + 1 \qquad \text{where} \qquad h(s) = 1 - \xi\left(\frac{s-\mu}{\sigma}\right)$$

Thus, as:

- $\lambda \to 1$, BCGEV → GEV distribution
- $\lambda \to 0$, BCGEV → GPD distribution (by L'Hopital's rule)

Benefit: GEV and GPD are now nested models and can be compared statistically.

Box-Cox GEV Distribution II

Application of the BCGEV model:

- **A high threshold is set - about 2 standard deviations above the mean of the parent data**
- **Data arranged sequentially: $s_1 \leq \ldots s_r \leq \ldots s_n$**

Estimation of BCGEV:

- **Maximum likelihood estimation not robust, so**
- **Non-linear regression estimation used:**

Residual

$$\log\left[\left(-\frac{1}{\lambda}\right)\log\left(1+\lambda\left(\frac{r}{n+1}-1\right)\right)\right]=\frac{1}{\xi}\log\left[1-\xi\left(\frac{s_r-\mu}{\sigma}\right)\right]+\eta$$

Minimize the sum of the squares of the residuals (SSR), $\Sigma\eta^2$

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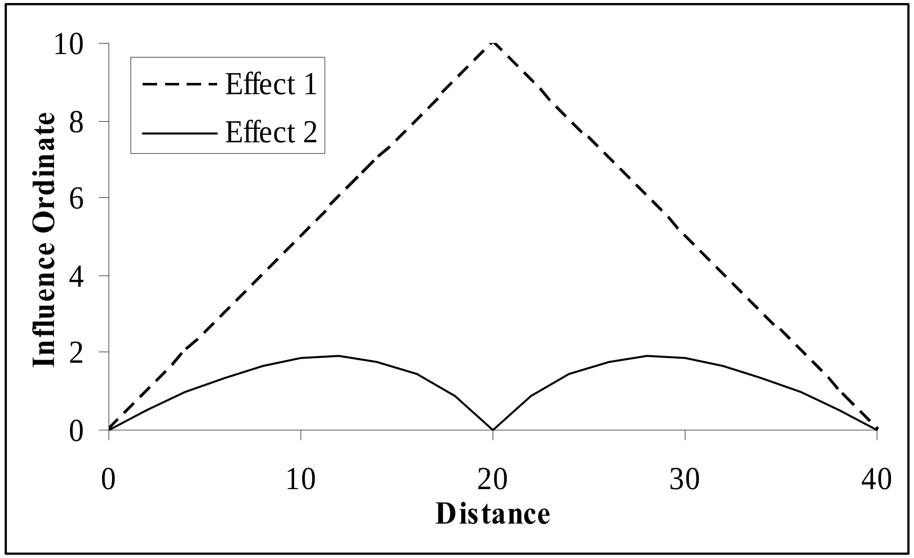
Bridge Traffic Loading I

- Using real traffic measured using Weigh-In-Motion
- Traffic characteristics are statistically modelled
- Monte Carlo simulation allows more traffic to be studied
- Load effects are calculated using influence lines of interest



Bridge Traffic Loading II

- 5 days of data from the A6 Paris-Lyon motorway is used as basis
- A 1000-day Monte Carlo traffic sample is generated
- Thus 1000 daily maximum static load effects
- Consider 5 bridge lengths of 20, 30, 40, 50, 60 m

3 load effects considered:

- LE1 – moment at B;
- LE2 – moment at E;
- LE3 – shear at A.

The optimal statistical extrapolation of this data set to determine lifetime load effect is what is considered in this work.

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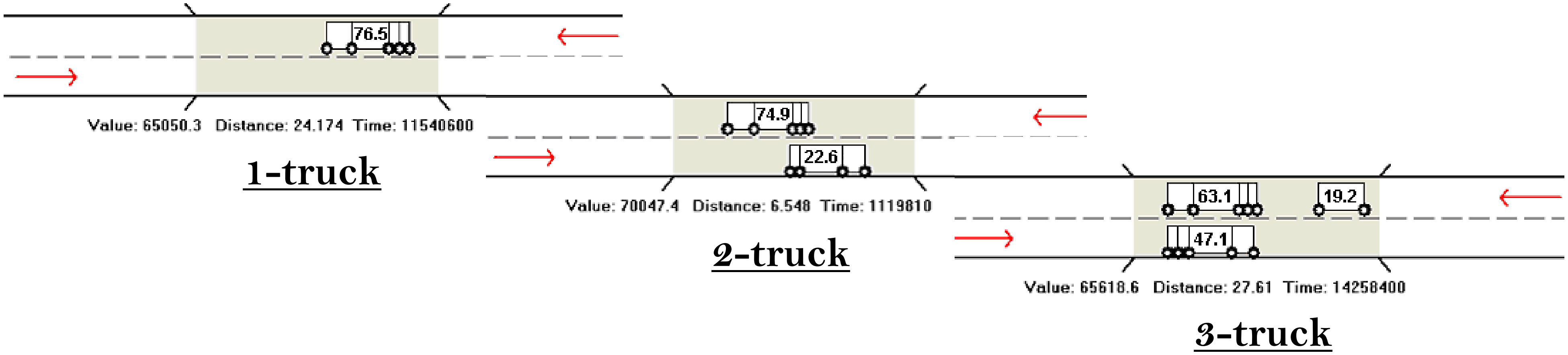
Bridge Traffic Loading III

In bridge traffic loading, different loading event types occur:



These loading events have different statistical distributions…

Use a composite distribution of load effect (Caprani et al 2008):

Composite Distribution ➡ $$G_C(z) = \prod_{i=1}^{N} G_i(z)$$ ⬅ Individual Event-type Distribution

Basis of BCGEV Analysis

For 3 load effects, 5 bridge lengths and each loading event type,

There are 41 data sets to be modelled.

11 thresholds are applied to the daily maximum data:

- In 0.5 standard deviation steps
- From $k = -2.5$ to $k = +2.5$ standard deviations about the mean

BCGEV model:

- Estimation of 'model parameter', λ is not robust
- Thus λ varied from 0 to 1 in 0.01 steps
- Best fit of remaining parameters then found for each λ.

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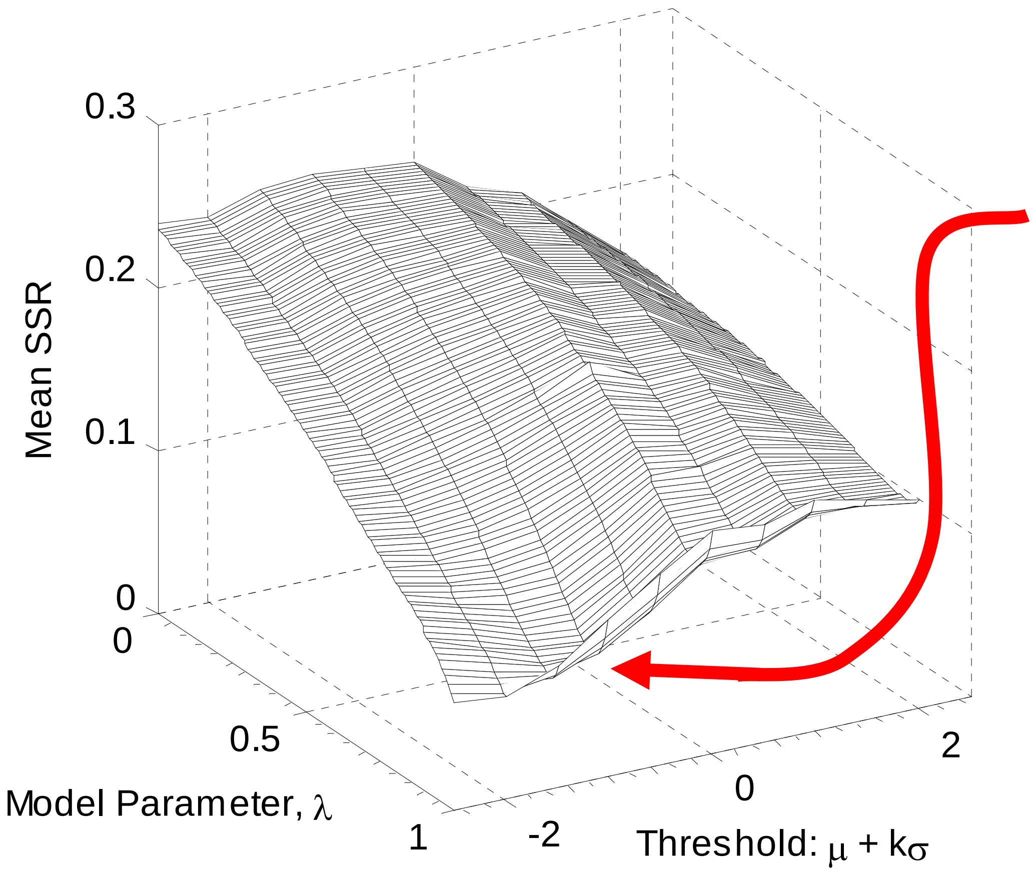
Optimum BCGEV Parameters

The mean SSR of the 41 data sets for each λ and threshold are taken to give:



Thus best fit on average is:

- Threshold, $k = -1.5$
- Model Parameter, $\lambda = 0.98$

Also:

- Best fit model parameter always $0.9 < \lambda < 1.0$
- Thus GEV better than GPD for bridge traffic loading?

Likelihood Ratio Test I

Using the LR test which applies to nested models:

- Determine of GEV or GPD (or neither) better represents the data
- Calculate the statistical significance of the representation

Calculate:

- Standard Error of Regression (SER) – the mean error per data point: SSR/n
- The LR statistic then is:

$$\mathrm{LR} = n\left(\log \mathrm{SER}_P - \log \mathrm{SER}_F\right)$$

Where:

- P – SER of partial model fit (GEV or GPD)
- F – SER of full model fit (BCGEV)

Likelihood Ratio Test II

This LR statistic is approx. χ^2- distributed with 1 degree of freedom:

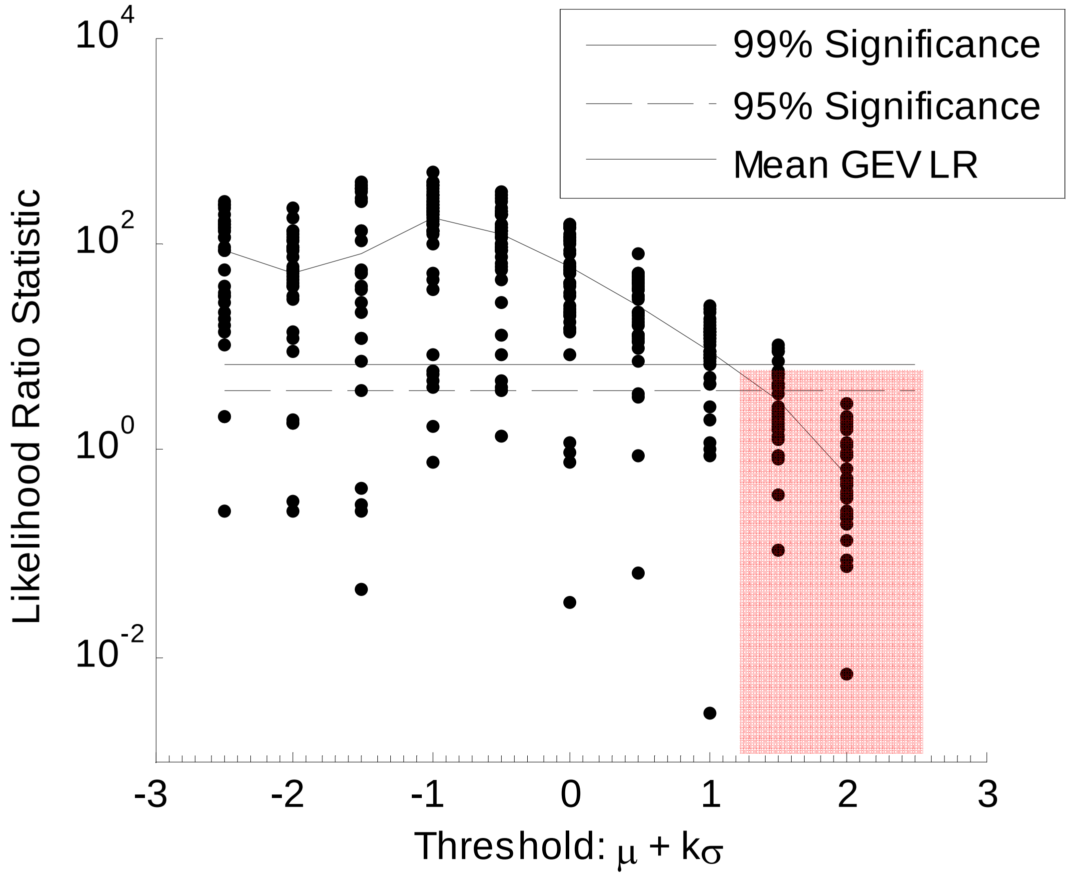
- For 95% significance level - critical value is 3.842
- For 99% significance level - critical value is 6.635

Hypothesis: partial model adequately represents data:

- Reject if LR statistic greater than critical value at chosen significance level

Significance Testing I

For the GEV distribution:



NB: Reject hypothesis if LR statistic > critical value

Thus:

- **GEV not statistically significant for most thresholds**
- **For about $k = +1.5$ and above, GEV is significant (shaded area)**

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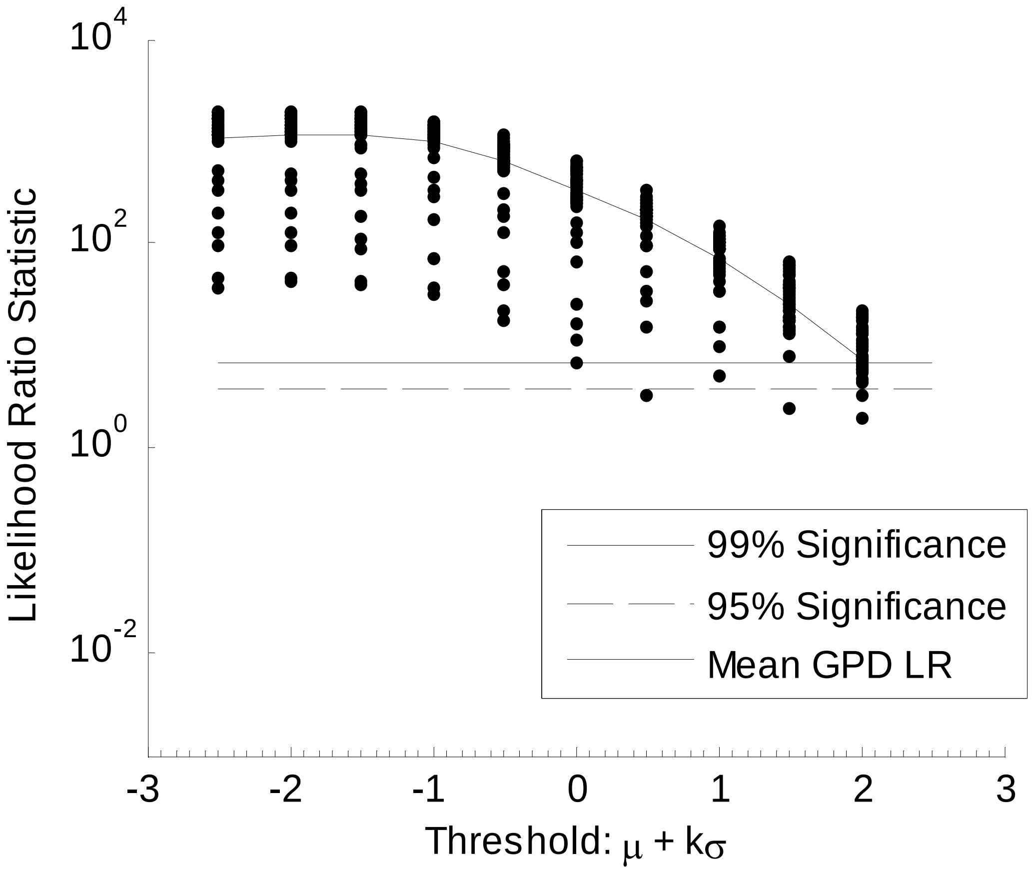
Significance Testing II

For the GPD distribution:



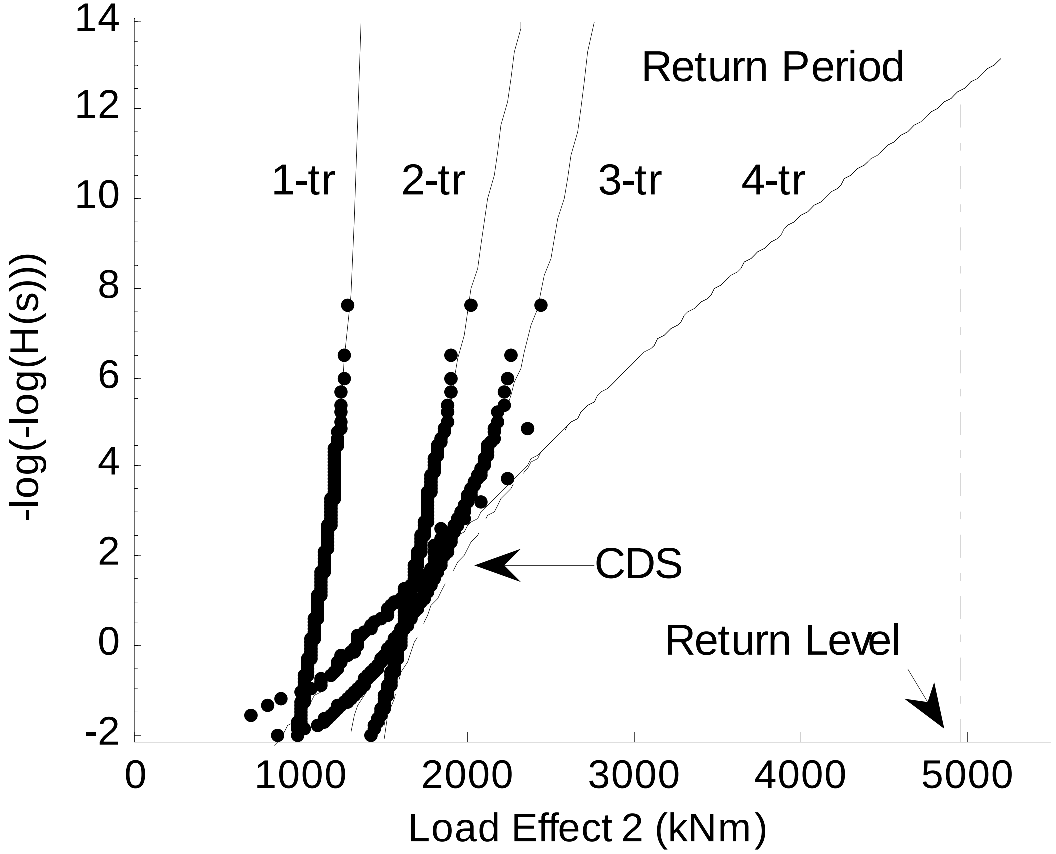
NB: Reject hypothesis if LR statistic > critical value

Thus:

- GPD not statistically significant for all thresholds

Load Effect Prediction I

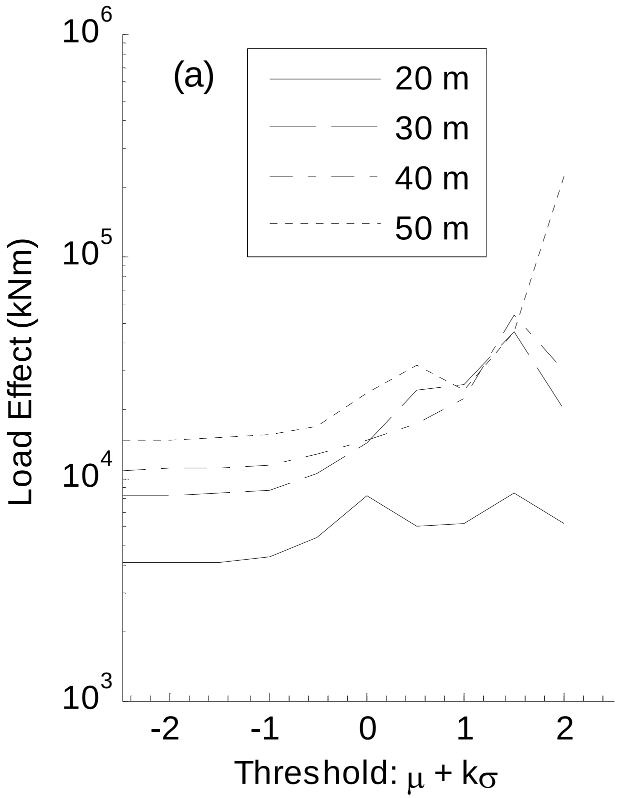
For each span and load effect, extrapolate the BCGEV fit:



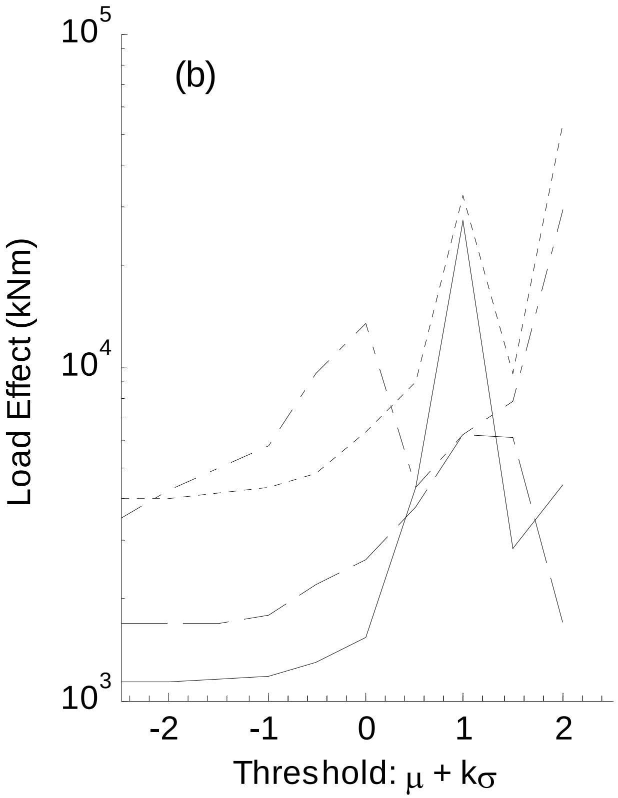
Bridge Length 40 m

Load Effect Prediction II

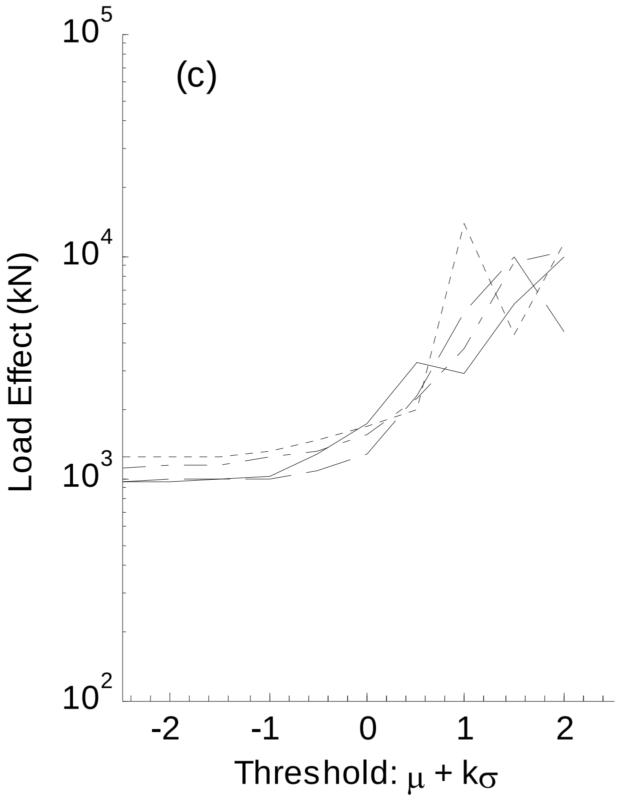
The BCGEV distribution predictions of lifetime load effect by threshold:



Load Effect 1



Load Effect 2



Load Effect 3

Load Effect Prediction III

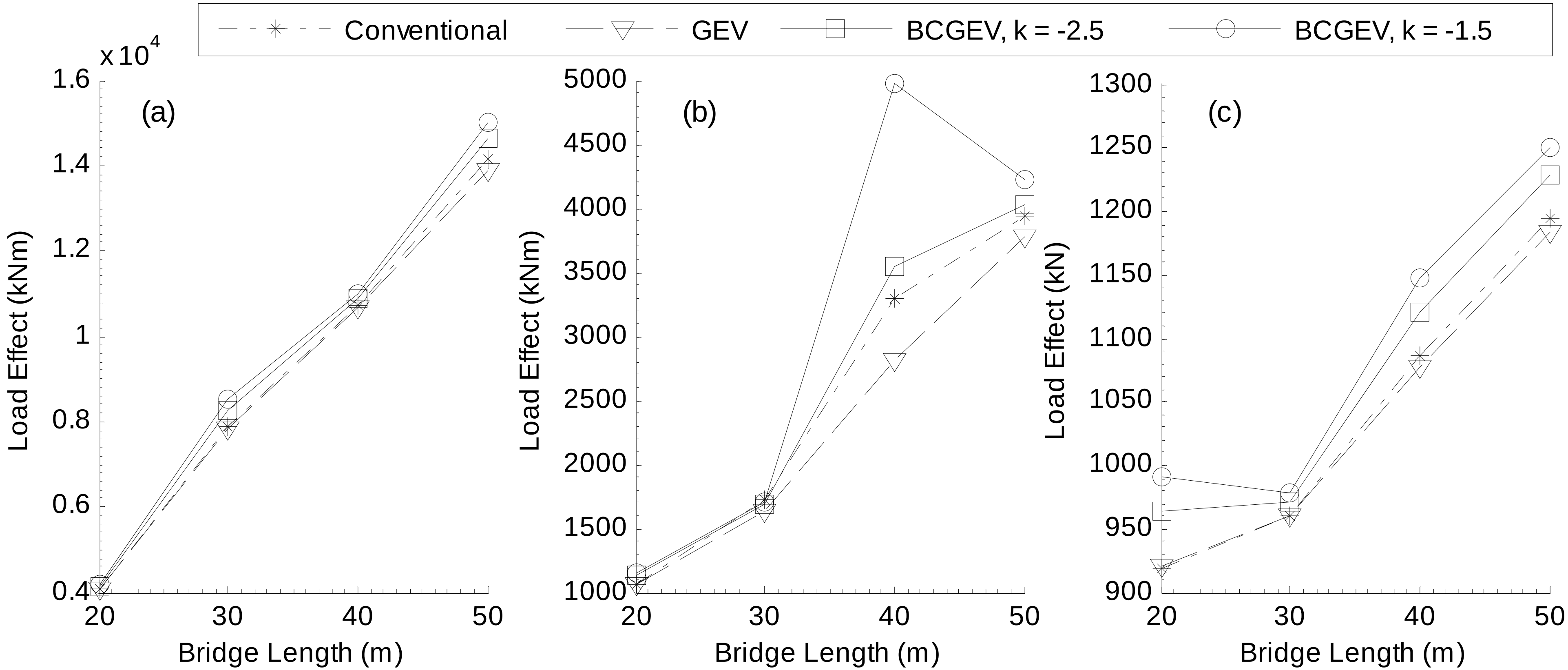
Comparison of different prediction methods:

- **Conventional: GEV model, ignoring different loading event types**
- **GEV: using CDS to account for different loading event types**
- **BCGEV, $k = -2.5$: considers all data and uses CDS**
- **BCGEV, $k = -1.5$: the 'global optimum' threshold identified previously**

Comparison with GPD not included as the best fit model parameter λ was never found to be close to zero for this data.

Load Effect Prediction IV

Comparison of different prediction methods:



Load Effect 1 **Load Effect 2** **Load Effect 3**

Conclusions I

- The Box-Cox-GEV model allows the data to determine the appropriate form of extreme value analysis.
- The BCGEV model has been extended with Composite Distribution Statistics (CDS) to account for the different loading event types.
- The BCGEV model is a better fit than the GEV and GPD models with considerable statistical significance, for almost all thresholds considered.
- Bridge traffic load effect data lies strongly in the domain of the GEV distribution.

Conclusions II

- An optimum threshold level to apply to daily maximum load effect has been identified, $k = -1.5$.
- The BCGEV model is stable for $k < 0$, i.e. thresholds below the mean daily maximum load effect.
- The BCGEV model gives slightly higher lifetime load effect predictions that other methods.
- The BCGEV model predictions were found to be more sensitive to different loading event types than other models.

Overall Conclusion:

The BCGEV model is more flexible and so more sympathetic to the data, giving increased confidence to load effect predictions.

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