Fixed-End Moments

Loading

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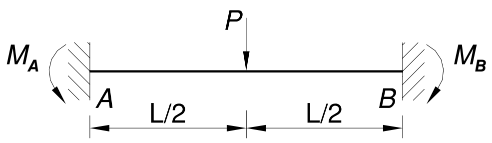
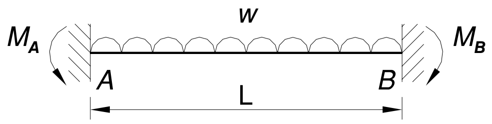
<tr><th>M_A</th><th>Configuration</th><th>M_B</th></tr>
<tr><td>$+\frac{PL}{8}$</td><td>
</td><td>$-\frac{PL}{8}$</td></tr>
<tr><td>$+\frac{wL^2}{12}$</td><td>
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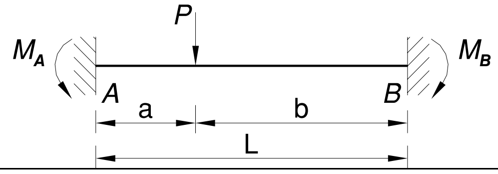
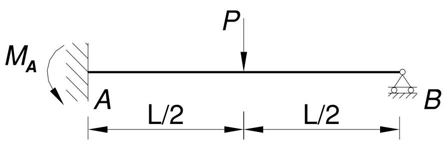
<tr><td>$+\frac{Pab^2}{L^2}$</td><td>
</td><td>$-\frac{Pa^2b}{L^2}$</td></tr>
<tr><td>$+\frac{3PL}{16}$</td><td>
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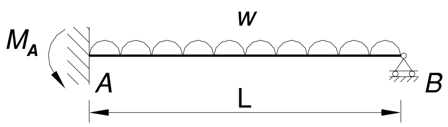
<tr><td>$+\frac{wL^2}{8}$</td><td>
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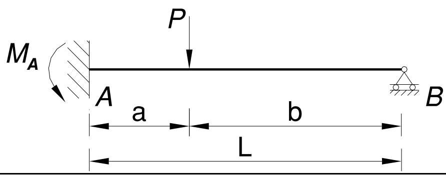
<tr><td>$+\frac{Pab\left(2L-a\right)}{2L^2}$</td><td>
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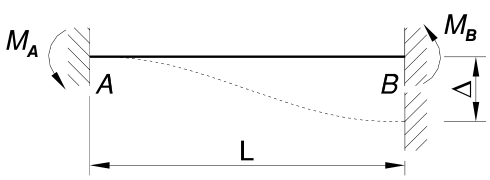
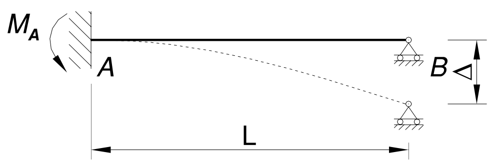
Displacements

<table>
<tr><th>M_A</th><th>Configuration</th><th>M_B</th></tr>
<tr><td>$+\frac{6EI\Delta}{L^2}$</td><td>
</td><td>$+\frac{6EI\Delta}{L^2}$</td></tr>
<tr><td>$+\frac{3EI\Delta}{L^2}$</td><td>
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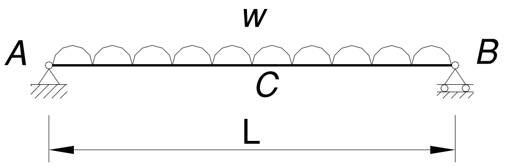
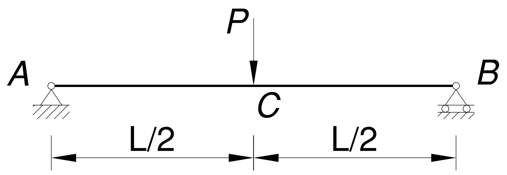
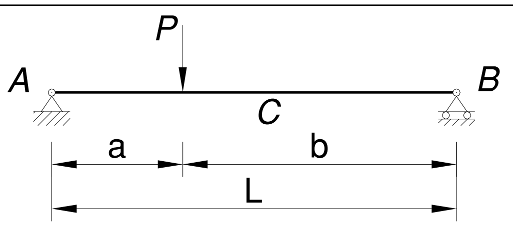
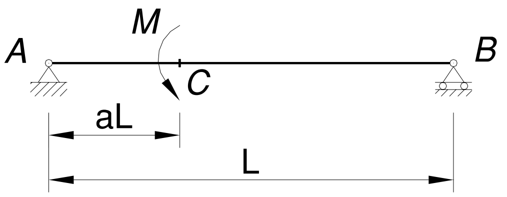
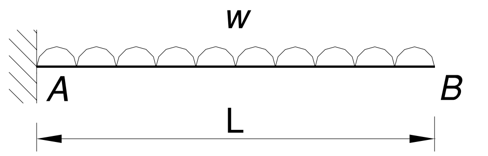
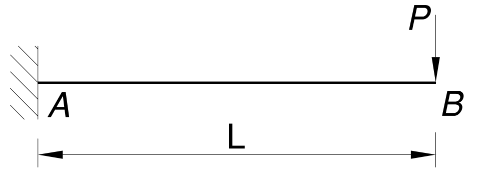
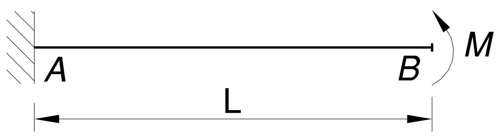
Displacements

Configuration	Translations	Rotations
	$\delta_C = \frac{5wL^4}{384EI}$	$\theta_A = -\theta_B = \frac{wL^3}{24EI}$
	$\delta_C = \frac{PL^3}{48EI}$	$\theta_A = -\theta_B = \frac{PL^2}{16EI}$
	$\delta_C \cong \frac{PL^3}{48EI}\left[\frac{3a}{L} - 4\left(\frac{a}{L}\right)^3\right]$	$\theta_A = \frac{Pa(L-a)}{6LEI}(2L-a)$ $\theta_B = -\frac{Pa}{6LEI}(L^2-a^2)$
	$\delta_C = \frac{ML^2}{3EI}a(1-a)(1-2a)$	$\theta_A = \frac{ML}{6EI}(3a^2-6a+2)$ $\theta_B = \frac{ML}{6EI}(3a^2-1)$
	$\delta_B = \frac{wL^4}{8EI}$	$\theta_B = \frac{wL^3}{6EI}$
	$\delta_B = \frac{PL^3}{3EI}$	$\theta_B = \frac{PL^2}{2EI}$
	$\delta_B = \frac{ML^2}{2EI}$	$\theta_B = \frac{ML}{EI}$

Matrix Stiffness Analysis

Truss Element Stiffness Matrix

$$K_{ij} = \begin{bmatrix} K11 & K12 \\ K21 & K22 \end{bmatrix}$$

$$K11 = \begin{bmatrix} \cos^2\alpha & \cos\alpha\cdot\sin\alpha \\ \cos\alpha\cdot\sin\alpha & \sin^2\alpha \end{bmatrix}$$

$$K11 = K22 = -K12 = -K21$$

Member Force

$$P_{ij} = \left(\frac{EA}{L}\right)_{ij} \begin{bmatrix} \cos\alpha & \sin\alpha \end{bmatrix} \begin{Bmatrix} \delta_{jx} - \delta_{ix} \\ \delta_{jy} - \delta_{iy} \end{Bmatrix}$$

Frame Element Stiffness Matrix

$$K_{ij} = \begin{bmatrix} K11 & K12 \\ K21 & K22 \end{bmatrix}$$

$$K11 = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \qquad K12 = K21 = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \end{bmatrix}$$

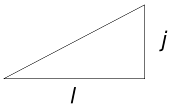
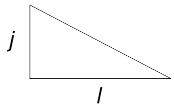
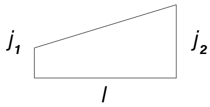
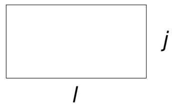
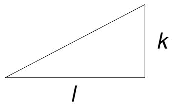
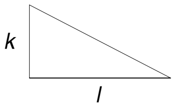
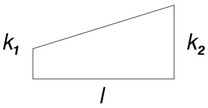
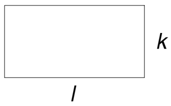
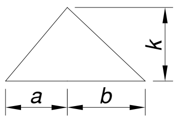
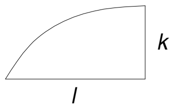
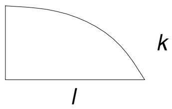
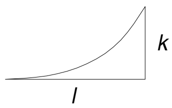
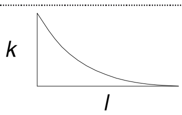
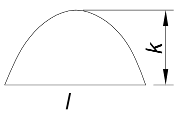
$$K22 = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Frame Element Transformation Matrix

$$T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Virtual Work

Volume Integrals

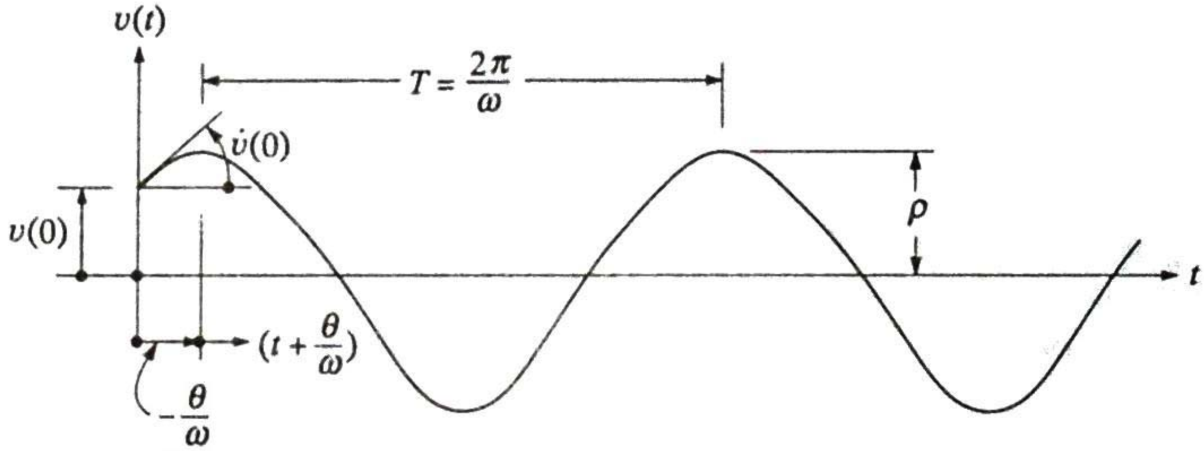
				
	$\frac{1}{3}jkl$	$\frac{1}{6}jkl$	$\frac{1}{6}(j_1+2j_2)kl$	$\frac{1}{2}jkl$
	$\frac{1}{6}jkl$	$\frac{1}{3}jkl$	$\frac{1}{6}(2j_1+j_2)kl$	$\frac{1}{2}jkl$
	$\frac{1}{6}j(k_1+2k_2)l$	$\frac{1}{6}j(2k_1+k_2)l$	$\frac{1}{6}\left[j_1(2k_1+k_2)+ j_2(k_1+2k_2)\right]l$	$\frac{1}{2}j(k_1+k_2)l$
	$\frac{1}{2}jkl$	$\frac{1}{2}jkl$	$\frac{1}{2}(j_1+j_2)kl$	jkl
	$\frac{1}{6}jk(l+a)$	$\frac{1}{6}jk(l+b)$	$\frac{1}{6}\left[j_1(l+b)+ j_2(l+a)\right]k$	$\frac{1}{2}jkl$
	$\frac{5}{12}jkl$	$\frac{1}{4}jkl$	$\frac{1}{12}(3j_1+5j_2)kl$	$\frac{2}{3}jkl$
	$\frac{1}{4}jkl$	$\frac{5}{12}jkl$	$\frac{1}{12}(5j_1+3j_2)kl$	$\frac{2}{3}jkl$
	$\frac{1}{4}jkl$	$\frac{1}{12}jkl$	$\frac{1}{12}(j_1+3j_2)kl$	$\frac{1}{3}jkl$
	$\frac{1}{12}jkl$	$\frac{1}{4}jkl$	$\frac{1}{12}(3j_1+j_2)kl$	$\frac{1}{3}jkl$
	$\frac{1}{3}jkl$	$\frac{1}{3}jkl$	$\frac{1}{3}(j_1+j_2)kl$	$\frac{2}{3}jkl$

Structural Dynamics

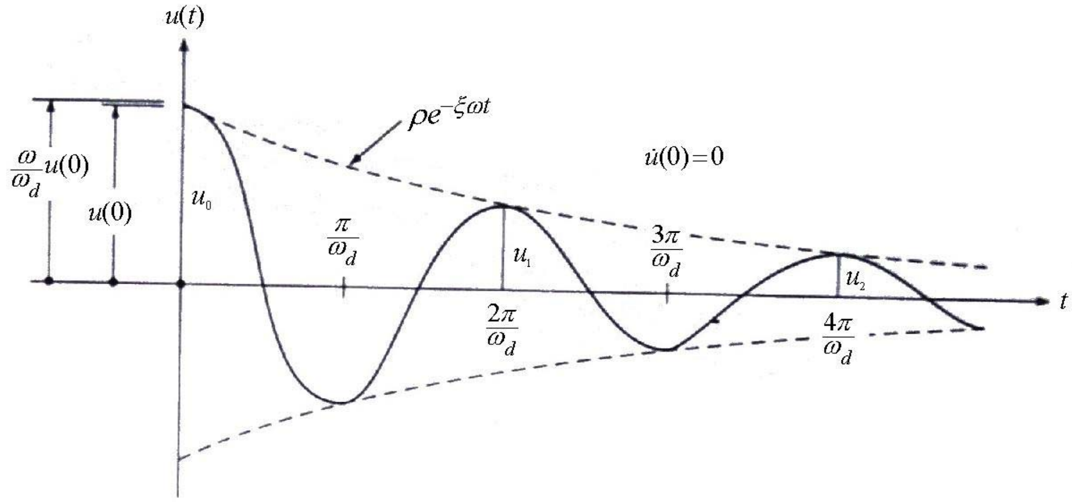
SDOF Systems

Fundamental equation of motion	$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=F(t)$
Equation of motion for free vibration	$\ddot{u}(t)+2\xi\omega\dot{u}(t)+\omega^2u(t)=0$
Relationship between frequency, circular frequency, period, stiffness and mass: Fundamental frequency for an SDOF system.	$f=\frac{1}{T}=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$
Coefficient of damping	$2\xi\omega=\frac{c}{m}$
Circular frequency	$\omega^2=\frac{k}{m}$
Damping ratio	$\xi=\frac{c}{c_{cr}}$
Critical value of damping	$c_{cr}=2m\omega=2\sqrt{km}$
General solution for free-undamped vibration	$u(t)=\rho\cos\left(\omega t+\theta\right)$ $\rho=\sqrt{u_0^2+\left(\frac{\dot{u}_0}{\omega}\right)^2}$; $\tan\theta=\frac{-\dot{u}_0}{u_0\omega}$
Damped circular frequency, period and frequency	$\omega_d=\omega\sqrt{1-\xi^2}$ $T_d=\frac{2\pi}{\omega_d}$; $f_d=\frac{\omega_d}{2\pi}$
General solution for free-damped vibrations	$u(t)=\rho e^{-\xi\omega t}\cos\left(\omega_d t+\theta\right)$ $\rho=\sqrt{u_0^2+\left(\frac{\dot{u}_0+\xi\omega u_0}{\omega_d}\right)^2}$; $\tan\theta=\frac{\xi\omega u_0-\dot{u}_0}{u_0\omega_d}$
Logarithmic decrement of damping	$\delta=\ln\frac{u_n}{u_{n+m}}=2m\pi\xi\frac{\omega}{\omega_d}$
Half-amplitude method	$\xi\cong\frac{0.11}{m}$ when $u_{n+m}=0.5u_n$

Amplitude after *p*-cycles	$u_{n+p} = \left(\frac{u_{n+1}}{u_n}\right)^p u_n$
Equation of motion for forced response (sinusoidal)	$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F_0 \sin \Omega t$
General solution for forced-damped vibration response and frequency ratio	$u_p(t) = \rho \sin(\Omega t - \theta)$ $\rho = \frac{F_0}{k}\left[\left(1-\beta^2\right)^2 + \left(2\xi\beta\right)^2\right]^{-1/2}$; $\tan\theta = \frac{2\xi\beta}{1-\beta^2}$ $\beta = \frac{\Omega}{\omega}$
Dynamic amplification factor (DAF)	$\text{DAF} \equiv D = \left[\left(1-\beta^2\right)^2 + \left(2\xi\beta\right)^2\right]^{-1/2}$



Undamped free-vibration response



General case of an under-critically damped system.