Structural Mechanics

Column Behaviour

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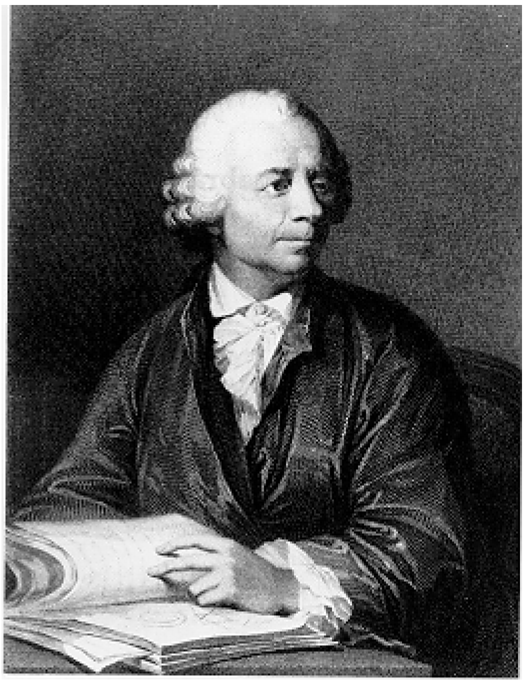
Contents

1. Introduction

1.1 Background

In the linear elastic analysis of structures, we have assumed that compression members are limited in load capacity in the same way that tension members are, by ensuring the yield stress of the material is not exceeded. However, as can easily be checked with a ruler, compression members often fail long before the material yields due to buckling. So our problem is to identify reduced stress limits that should apply for compression members so that buckling does not occur.

The first person to study this problem was Euler ('oil-er') as a means to demonstrate his ability to solve differential equations. Some of the important results in buckling retain his name.



Leonhard Euler (1707 – 1783)

1.2 Stability of Equilibrium

A structure will be in an initial equilibrium position. The stability of its equilibrium can be assessed by examining the structure's behaviour in an adjacent position. There are three states:

- **Stable equilibrium**: the structure tends to return to its initial position. This is the best situation to have structures in.
- **Neutral (or critical) equilibrium**: the structure moves to a displaced configuration and remains in that position. This does not make for good structure.
- **Unstable equilibrium**: any movement from the initial position causes further movement resulting in a 'runaway' failure of the structure.

In relation to columns, the stability of equilibrium takes the form:

- **Stable**: deflections to not result in extra bending moments, and hence extra deflections.
- **Critical** $P = P_{cr}$: the load is at a critical value where the column remains in any displaced position.
- **Unstable** $P > P_{cr}$: the load is greater than the critical load and so divergent displacements occur, leading to failure.

These situations look like this:

2. Buckling Solutions

2.1 Introduction

A perfect column (perfectly straight) is one which is perfectly straight and so carries axial load up to the yield stress of the material. Since in reality columns are not perfectly straight, buckling occurs:

In our solutions for buckling, we will find that both the perfectly straight and buckled profiles are both possible theoretically. However, since it is the real behaviour that is of interest, we will focus on the buckled solutions.

2.2 Pinned-Pinned Column

Formulation

Firstly, consider the buckled configuration of a pin-ended column and draw a free body diagram of part of the column:

From this we see:

$$M = -Py$$

So for equilibrium:

$$M + Py = 0$$

We know from Euler-Bernoulli bending theory that:

$$M = EI\frac{d^2y}{dx^2}$$

And so we have:

$$EI\frac{d^2y}{dx^2} + Py = 0 \tag{1}$$

Dividing across by *EI* gives:

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

If we make the substitution:

$$k^2 = \frac{P}{EI} \tag{2}$$

We then have:

$$\frac{d^2y}{dx^2} + k^2y = 0 \tag{3}$$

This is a second-order linear homogenous differential equation in y. We seek a solution for y which will be some function of x. The Appendix shows that the general solution to this equation is:

$$y = A\cos kx + B\sin kx \tag{4}$$

where A and B are constants to be evaluated from the boundary conditions of the problem.

Relevant Solution

To get the particular solution to our problem, we know that we have no deflection at the pinned end, that is:

$$\text{At } x = 0, y = 0$$

Substituting this into equation (4):

$$0 = A\cos k0 + B\sin k0$$

Since $\cos(0) = 1$ and $\sin(0) = 0$, we have:

$$\begin{aligned} &0 = A(1) + B(0) \\ &A = 0 \end{aligned}$$

Thus equation (4) becomes:

$$y = B\sin kx \tag{5}$$

Using the second boundary condition, at $x = L, y = 0$, we have:

$$0 = B\sin kL \tag{6}$$

There are two possibilities now. The first is $B = 0$ which makes $y = 0$ by equation (5) . This means that a possible solution is for no buckling to occur, in other words, the perfect column. Since we know that this is highly unlikely, and that buckling doesn't occur, we must consider the other possibility from equation (6):

$$\sin kL = 0 \tag{7}$$

We know that this only happens at values of:

$$\begin{aligned} kL &= 0, \pi, 2\pi, 3\pi, \ldots \\ &= n\pi \end{aligned}$$

where $n = 0,1,2,3,\ldots$. Therefore we have:

$$k = \frac{n\pi}{L} \tag{8}$$

So from equation (2) we have:

$$k^2 = \frac{P}{EI} = \frac{n^2\pi^2}{L^2}$$

And so the critical loads at which the column buckles are:

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2} \tag{9}$$

Further, by using equation (8) in equation (5) the buckled shape is got as:

$$y = B\sin\frac{n\pi}{L}x \tag{10}$$

Euler Buckling Load

Since we are interested in the lowest load that the column will buckle at, we use the value $n = 1$ to find the Euler Buckling Load, P_E, as:

$$P_E = \frac{\pi^2 EI}{L^2} \tag{11}$$

And we also find the displaced shape from equation (10) as:

$$y = B \sin \frac{\pi}{L} x \tag{12}$$

This defines a half sine-wave curve as being the buckled shape of the column. Notice that we have no information about *B*, the amplitude of the displacement. This is because the column is in neutral equilibrium at P_E and will be in equilibrium at any displacement amount.

Other Buckling Modes

In general we can see that the column can buckle in the shapes:

n	0	1	2	3	…
Critical Load $P_{cr} =$	Infinite	$P_E = \frac{\pi^2 EI}{L^2}$	$\frac{4\pi^2 EI}{L^2}$	$\frac{9\pi^2 EI}{L^2}$	$\frac{n^2\pi^2 EI}{L^2}$
Mode Shape $y =$	0	$y = B\sin\frac{\pi}{L}x$	$B\sin\frac{2\pi}{L}x$	$B\sin\frac{3\pi}{L}x$	$B\sin\frac{n\pi}{L}x$
Plot					

However, to achieve these other buckling loads, the lower modes must be prevented from occurring by lateral restraints:

Critical Stress

For design, we are interested in the stress that the material undergoes at the time of buckling – the critical stress, σ_{cr}:

$$\sigma_{cr} = \frac{P_E}{A} = \frac{\pi^2 EI}{L^2 A} \tag{13}$$

Looking at the factor I/A, we see that it is a property of the shape of the cross section, and is in units of $[\text{length}]^4 / [\text{length}]^2 = [\text{length}]^2$. Therefore, we define a new geometric property, *r*, called the **Radius of Gyration** as:

$$r = \sqrt{\frac{I}{A}} \tag{14}$$

And so *r* has units of length. The radius of gyration can be thought of as a distance from the centroid at which the area of the cross section is concentrated for calculating the second moment of area, *I*, since by (14),

$$I = Ar^2 \tag{15}$$

The critical stress can now be expressed as:

$$\sigma_{cr} = \frac{\pi^2 E r^2}{L^2}$$

And we can see that the dimensional properties of the column are summed up by the factor r^2/L^2, which represents a ratio of *r* to *L*. Thus we define the **Slenderness Ratio**, λ, as:

$$\lambda = \frac{L}{r} \tag{16}$$

Finally then, we have the equation for critical stress as:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{17}$$

A plot of the critical stress against slenderness is called a **strut curve** and looks like:

As can be seen, at low slendernesses (that is short stocky columns), the critical stress (to cause bucking) reaches very high values. Since the maximum stress in the material is the yield stress, me must cap the curve at σ_y.

Finally, notice that typical experimental results fall below the Euler strut curve. This is because the theory examined so far is for perfectly straight columns that have somehow begun to buckle. In real columns there will be some initial imperfections which have the effect of reducing the strength of the column. These initial imperfections can be represented by an initial displacement curve.

2.3 Column with Initial Displacements

Problem Formulation

The imperfections in the manufacture of real columns mean that an initial displacement curve exists in the column, prior to loading. Since any curve can be represented by a Fourier series expansion, we will approximate the initial displaced shape by the first term of a Fourier series – a sine curve, the equation of which is:

$$y_0\left(x\right) = a\sin\frac{\pi x}{L} \tag{18}$$

where, at the midpoint of the column, the initial displacement is *a*.

Considering a free-body diagram as before:

gives the equilibrium equation as:

$$EI\frac{d^2y}{dx^2}+P\left(y+y_0\right)=0$$

Thus we have:

$$\frac{d^2y}{dx^2}+\frac{P}{EI}y+\frac{P}{EI}y_0=0$$

Using equation (2) gives:

$$\frac{d^2y}{dx^2} + k^2 y + k^2 y_0 = 0$$

And so using equation (18) we have

$$\frac{d^2y}{dx^2} + k^2 y = -k^2 a \sin\frac{\pi x}{L} \tag{19}$$

This is a non-homogenous second order differential equation.

Solution

The solution to non-homogenous differential equations is made up of two parts:

- The **complimentary solution** (denoted y_C): this is the solution to the corresponding homogenous equation. That is, the solution when the right hand side is zero. We have this from before as equation (4):

$$y_C = A\cos kx + B\sin kx \tag{20}$$

- The **particular solution** (denoted y_P): for the function on the right hand side, the solution is verified in the Appendix as:

$$\begin{aligned} y_P &= \frac{-k^2 a}{-\left(\frac{\pi}{L}\right)^2 + k^2}\sin\frac{\pi x}{L} \\ &= \frac{k^2 a}{\frac{\pi^2}{L^2} - k^2}\sin\frac{\pi x}{L} \end{aligned} \tag{21}$$

Thus the total solution is:

$$\begin{aligned} y &= y_C + y_P \\ &= A\cos kx + B\sin kx + \frac{k^2 a}{\frac{\pi^2}{L^2} - k^2}\sin\frac{\pi x}{L} \end{aligned} \tag{22}$$

To find the constants, we know that for the pinned-pinned column, $y = 0$ at $x = 0$:

$$y(0) = A\cos k(0) + B\sin k(0) + \frac{k^2 a}{\frac{\pi^2}{L^2} - k^2}\sin\frac{\pi}{L}(0)$$

$$0 = A$$

Also, $y = 0$ at $x = L$, giving:

$$y(L) = B\sin k(L) + \frac{k^2 a}{\frac{\pi^2}{L^2} - k^2}\sin\frac{\pi}{L}(L)$$

$$0 = B\sin kL$$

Although this is the same equation as found for the perfectly straight column, we must consider the implications. If $B \neq 0$ then $\sin kL = 0$ and so $kL = \pi$ as before. This yields $k = \pi/L$, or $k^2 = \pi^2/L^2$. Substituting this into equation (22) means that the third term is infinite and so the deflection is infinite. Since this is impossible for a stable column with $P < P_{cr}$, we conclude that $B = 0$ and we are left with:

$$y = \frac{k^2 a}{\frac{\pi^2}{L^2} - k^2}\sin\frac{\pi x}{L} \tag{23}$$

This equation represents the deflections of the column caused by the loading. The total deflection will be that caused by the loading, in addition to the initial imperfection deflection curve:

$$y_{tot} = y + y_0$$

And so:

$$y_{tot} = \frac{k^2}{\frac{\pi^2}{L^2} - k^2} a \sin\frac{\pi x}{L} + a \sin\frac{\pi x}{L}$$

Solving out, and dropping the *tot* subscript on *y* gives::

$$y = \left(\frac{k^2}{\pi^2 / L^2 - k^2} + 1 \right) a \sin\frac{\pi x}{L}$$

And so:

$$y = \left(\frac{\pi^2 / L^2}{\pi^2 / L^2 - k^2} \right) a \sin\frac{\pi x}{L} \tag{24}$$

Now, using the expression for P_E (equation (11)), we have:

$$\frac{\pi^2}{L^2} = \frac{P_E}{EI}$$

And with the expression for k^2 (equation (2)), equation (24) becomes:

$$y = \frac{P_E / EI}{P_E / EI - P / EI} a \sin\frac{\pi x}{L}$$

And so we have:

$$y = \left[\frac{P_E}{P_E - P} \right] a \sin\frac{\pi x}{L} \tag{25}$$

The term in brackets thus amplifies the initial deflection, depending on how close we are to the critical buckling load. A plot of load against deflection shows:

Maximum Stress Consideration

At the mid-height of the column, the deflection will be largest, and thus so will the bending moment. The deflection at the mid-height is got from equation (25), with $x = L/2$:

$$y\left(\frac{L}{2}\right) = \left[\frac{P_E}{P_E - P}\right] a \sin\frac{\pi}{L}\left(\frac{L}{2}\right)$$
$$= \left[\frac{P_E}{P_E - P}\right] a$$

We can equally interpret this equation in terms of stresses by dividing each of the *P*s by *A*:

$$y\left(\frac{L}{2}\right) = \left[\frac{\sigma_E}{\sigma_E - \sigma}\right] a \tag{26}$$

where σ_E is the stress associated with the critical Euler load (equation (13)).

Consider again the free-body diagram of the column from mid-height to pin. There are two sources of stress:

1. The stresses due to the moment alone are:

$$\sigma_{\text{Moment}} = \frac{Mz}{I}$$

where *z* is the distance from the neutral axis of the fibre under consideration.

2. The stresses due to the axial force are:

$$\sigma_{\text{Axial}} = \frac{P}{A}$$

Thus the stresses at any point are given by:

$$\sigma = \frac{P}{A} + \frac{Mz}{I} \qquad (27)$$

Superposition of the stress diagrams shows this:

The maximum stress is on the outside face and is thus:

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} \qquad (28)$$

where c is the distance from the neutral axis to the inside face of the column.

Next, into equation (28), we introduce the relevant properties of equation (15) and the fact that $M = Py\left(L/2\right)$ yields:

$$\sigma_{\max} = \sigma + \frac{Pcy\left(L/2\right)}{Ar^2}$$

But, from equation (26) we know the displacement at mid height, $y\left(L/2\right)$:

$$\sigma_{\max} = \sigma + \frac{Pc}{Ar^2}\left[\frac{\sigma_E}{\sigma_E - \sigma}\right]a$$

At failure the maximum stress is the yield stress, σ_y. The stress associated with the load *P* when this occurs is σ_{cr}. Hence the governing equation becomes:

$$\sigma_y = \sigma_{cr} + \sigma_{cr}\frac{c}{r^2}\left[\frac{\sigma_E}{\sigma_E - \sigma_{cr}}\right]a$$

Giving:

$$\sigma_y = \sigma_{cr}\left[1 + \frac{ac}{r^2}\left(\frac{\sigma_E}{\sigma_E - \sigma_{cr}}\right)\right] \qquad (29)$$

We are looking to find the value of σ_{cr} that solves this equation. At the load corresponding to σ_{cr} failure occurs. As can be seen, this failure stress is a function of the section (through *r* and *c*) and the initial imperfection, *a*, as well as the usual Euler buckling load for the column (through σ_E).

Critical Stress for Buckling

To solve equation (29) for σ_{cr} we proceed as follows:

$$\sigma_y = \sigma_{cr}\left[1 + \frac{ac}{r^2}\left(\frac{\sigma_E}{\sigma_E - \sigma_{cr}}\right)\right]$$

$$\sigma_y r^2\left(\sigma_E - \sigma_{cr}\right) = \sigma_{cr} r^2\left(\sigma_E - \sigma_{cr}\right) + ac\sigma_{cr}\sigma_E$$

$$\sigma_y\sigma_E r^2 - \sigma_y\sigma_{cr} r^2 = \sigma_{cr}\sigma_E r^2 - \sigma_{cr}\sigma_{cr} r^2 + ac\sigma_{cr}\sigma_E$$

$$\sigma_y\sigma_E r^2 - \sigma_y\sigma_{cr} r^2 - \sigma_{cr}\sigma_E r^2 + \sigma_{cr}\sigma_{cr} r^2 - ac\sigma_{cr}\sigma_E = 0$$

$$\sigma_{cr}^2\left(r^2\right) + \sigma_{cr}\left(-\sigma_y r^2 - \sigma_E r^2 - ac\sigma_E\right) + \sigma_y\sigma_E r^2 = 0$$

$$\sigma_{cr}^2 + \sigma_{cr}\left(-\sigma_y - \sigma_E - \frac{ac}{r^2}\sigma_E\right) + \sigma_y\sigma_E = 0$$

We call the parameter that accounts for the initial imperfections called the **Perry Factor**:

$$\eta = \frac{ac}{r^2} \tag{30}$$

And this gives:

$$\sigma_{cr}^2 + \sigma_{cr}\left[-\sigma_y - \sigma_E\left(1+\eta\right)\right] + \sigma_y\sigma_E = 0 \tag{31}$$

This is a quadratic equation in σ_{cr} and so is solved by the usual:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:

$$a = 1$$
$$b = \left[-\sigma_y - \sigma_E \left(1 + \eta\right) \right]$$
$$c = \sigma_y \sigma_E$$

And this gives:

$$\sigma_{cr} = \frac{\sigma_y + \sigma_E \left(1 + \eta\right)}{2} - \left\{ \left[\frac{\sigma_y + \sigma_E \left(1 + \eta\right)}{2} \right]^2 - \sigma_y \sigma_E \right\}^{0.5} \tag{32}$$

Notice that we have chosen the lower root of the two possible solutions.

Equation (32) is called **the Perry-Robertson formula** and it gives the buckling stress in terms of the yield stress and initial imperfections of the column, as well as its Euler buckling load.

It is useful to introduce the following:

$$\phi = \frac{\sigma_y + \sigma_E \left(1 + \eta\right)}{2} \tag{33}$$

And so the Perry-Robertson formula (equation (32)) becomes:

$$\sigma_{cr} = \phi - \sqrt{\phi^2 - \sigma_y \sigma_E} \tag{34}$$

2.4 The Effective Length of Columns

So far we have only considered pinned-end columns but clearly other end conditions are possible. Analysis of the buckling loads for such columns can be carried out along the same lines as for pinned-end columns. However, a significant advantage can be got by remembering that a point of contraflexure (zero moment) behaves as a pin (zero moment). The distance between points of contraflexure can be considered as a pin-pin column, but with a smaller length than the overall column. We call this length the **effective length**, L_E. Analysis then proceeds as for pinned-end columns, but using the effective, rather than actual, length.

The effective lengths of some typical columns are:

Non-Sway Modes		
Pin-Pin	**Fix-Pin**	**Fix-Fix**

Sway Modes		
Fix-fix	**Fix-roller**	**Cantilever (fix-free)**

Notice from the above that the locations of the points of contraflexure do not have to be and can be located outside the column. That is, the column is buckling over a notional length of L_E.

The effective length only affects the slenderness and so the general case for slenderness is:

$$\lambda = \frac{L_E}{r} \tag{35}$$

3. Column Design

3.1 Background to BS5950

Initial Imperfections

Robertson performed many tests on struts to arrive at a suitable value for the initial imperfections in the approach outlined in the previous section. He suggested:

$$\eta = 0.003\lambda \tag{36}$$

where λ is the slenderness of the column, given by equation (16). More recently, the initial imperfection has been taken as:

$$\eta = 0.3\left(\frac{\lambda}{100}\right)^2 \tag{37}$$

The idea of linking the initial imperfections to the slenderness is intuitively appealing – the slimmer a column is, the more likely it is to have imperfections.

The steel design code BS5950 is based on the following:

$$\eta = \frac{a\left(\lambda - \lambda_0\right)}{1000} \tag{38}$$

In which:

- a is the Robertson constant (and is not the same as the a we had for the deflection of the column previously);
- λ_0 is called the limiting slenderness as is given by:

$$\lambda_0 = 0.2\sqrt{\frac{\pi^2 E}{\sigma_y}} \tag{39}$$

By rearranging equation (17) it can be seen that this is:

$$\lambda_0 = 0.2\lambda_{cr}$$

where λ_{cr} is the slenderness at which the Euler stress reaches the yield stress of the material.

As can be seen, the higher the value of *a*, the more initial imperfection is accounted for and the compressive strength reduces as a result.

Code Expressions

In BS5950, the Perry-Robertson formula is given in a slightly different form to that presented in equation (34). To get the code expression, we multiply top and bottom of equation (34) by $\phi + \sqrt{\phi^2 - \sigma_y \sigma_E}$ to get:

$$\sigma_{cr} = \frac{\left[\phi - \sqrt{\phi^2 - \sigma_y \sigma_E}\right]\left[\phi + \sqrt{\phi^2 - \sigma_y \sigma_E}\right]}{\phi + \sqrt{\phi^2 - \sigma_y \sigma_E}}$$

And multiplying out gives:

$$\begin{aligned} \sigma_{cr} &= \frac{\phi^2 - \phi^2 + \sigma_y \sigma_E}{\phi + \sqrt{\phi^2 - \sigma_y \sigma_E}} \\ &= \frac{\sigma_y \sigma_E}{\phi + \sqrt{\phi^2 - \sigma_y \sigma_E}} \end{aligned}$$

Lastly, to get the code expression, we must use the code notation which is:

$$\begin{aligned} p_C &\equiv \sigma_{cr} \\ p_y &\equiv \sigma_y \\ p_E &\equiv \sigma_E \end{aligned}$$

So finally we have the expression in Appendix C of BS5950:

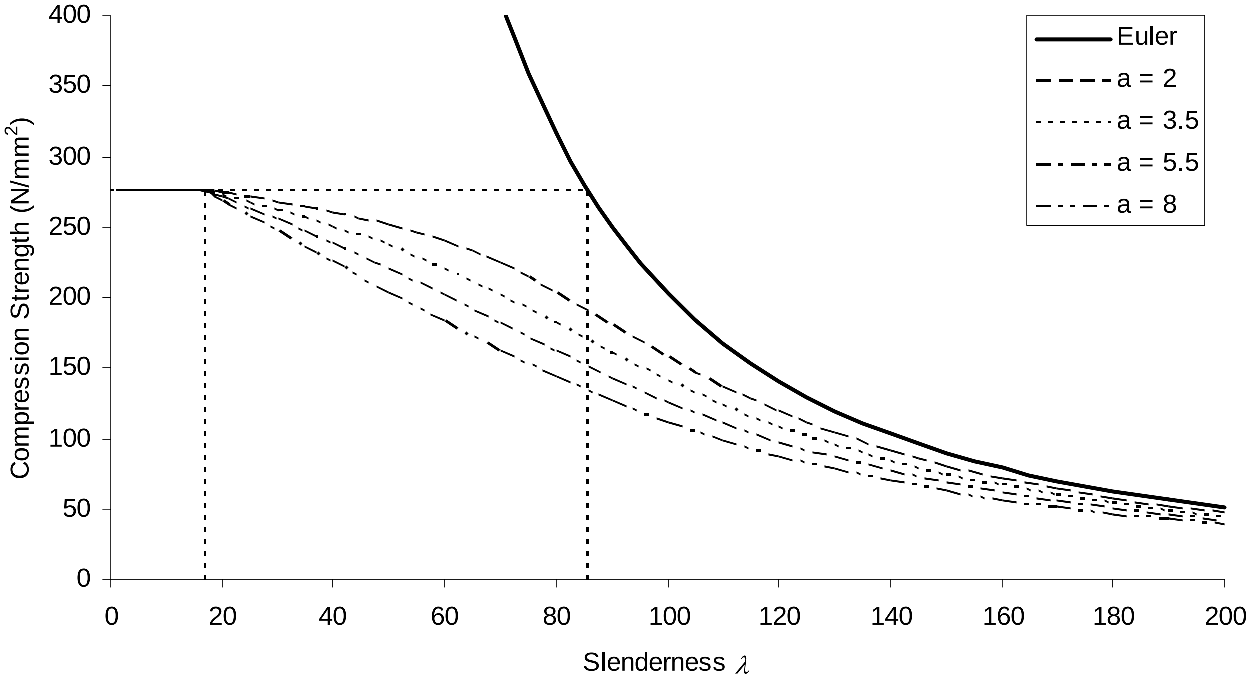
$$p_c = \frac{p_E p_y}{\phi + \sqrt{\phi^2 - p_E p_y}} \tag{40}$$

Strut Curves

BS5950 provides four values of the Robertson constant that may be used in design. It also specifies what value of *a* to use for the various types of steel section and the axis about which buckling may occur. The values are:

- $a = 2.0$ - strut curve (a);
- $a = 3.5$ - strut curve (b);
- $a = 5.5$ - strut curve (c);
- $a = 8.0$ - strut curve (d).

Given the initial imperfection model of equation (38) as well as the Perry-Roberston formula, equation (40), we can plot the four strut curves used in the code:



Also shown in this plot are the limiting slenderness, and the critical slenderness, discussed in relation to equation (38).

The code provides four tables (Table 24(a) to 24(d)) – corresponding to the strut curves, which are formatted as follows:

	p_y				
λ	225	.	.	.	275
15	.	.	.	.	.
.					
.	Values of p_c from equation (40)				
.					
350	.	.	.	.	.

Table 23 of the code allocates the strut curves to different section types and axes:

Effective Lengths

Table 22 of the code specifies the appropriate effective lengths for columns with different end conditions, end conditions that occur in practice. The meanings of the phrases in Table 22 are as follows:

Term	**Diagram**
Effectively held in position, not restrained in direction. (pin)	
Effectively held in position, partially restrained in direction. (rotational spring)	
Effectively held in position and restrained in direction. (fixed)	
Not held in position, and effectively restrained in direction. (fixed with sway)	
Not held in position, partially restrained in direction. (rotational spring with sway)	
Not held in position or restrained in direction. (free)	

3.2 Column Design Examples

Example 1

Problem

A 5.6 m high column consists of a 203 × 203 × 46 UC section. It is supported along its *x*-axis and is pinned at both ends. Find the buckling load.

Solution

Firstly, sketch the column:

Since the column is supported along its *x-x* axis, it can only buckle about its *y-y* axis. The relevant section properties for a 203 × 203 × 46 UC are:

Cross-Sectional Area	58.8 cm^2
Yield stress	275 N/mm^2
Modulus of Elasticity	205 kN/mm^2
Radius of gyration about the *y-y* axis	5.12 cm
Robertson Constant for the *y-y* axis	5.5

Thus:

$$\lambda = \frac{5600}{51.1} = 109.6 \approx 110$$

From Table 23 we see that we are using strut curve (c) and so $a = 5.5$. Also, $E = 205 \text{ kN/mm}^2$ and $p_y = 265 \text{ N/mm}^2$ from Table 9. Thus:

The limiting slenderness (equation (39)) is:

$$\lambda_0 = 0.2\sqrt{\frac{\pi^2 E}{p_y}} = 0.2\sqrt{\frac{\pi^2 \cdot 205 \times 10^3}{265}} = 17.48$$

The Perry Factor (equation (38)) is:

$$\eta = \frac{a(\lambda - \lambda_0)}{1000} = \frac{5.5(110 - 17.48)}{1000} = 0.509$$

The Euler stress (equation (17) is:

$$p_E = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \cdot 205\times10^3}{110^2} = 168 \text{ N/mm}^2$$

The modifying stress (equation (33)) is:

$$\phi = \frac{p_y + (\eta+1)\, p_E}{2} = \frac{265 + (0.509+1)168}{2} = 259 \text{ N/mm}^2$$

And so the compressive strength (equations (34)or (40)):

$$p_c = \frac{p_E\, p_y}{\phi + \sqrt{\phi^2 - p_E\, p_y}} = \frac{168\cdot 265}{259 + \sqrt{259^2 - 168\cdot 265}} = 108.6 \text{ N/mm}^2$$

Thus the buckling load is:

$$P = A_g\, p_c = \frac{5880\cdot 108.6}{10^3} = 640.4 \text{ kN}$$

To check this, use Table 24(c), for $\lambda \geq 110$ and $p_y = 265 \text{ N/mm}^2$ gives: $p_c = 108 \text{ N/mm}^2$ and so the capacity is $108\cdot 5880/10^3 = 635 \text{ kN}$, which is similar to the previous calculation.

Example 2

Problem

For the column of Example 1, the restraint along the *x*-*x* axis has to be removed. Determine the buckling capacity.

Solution

Again, sketch the column:

Since the column can now buckle about both its *x*-*x* axis and *y*-*y* axis, we will need to determine the buckling capacity of each axis. Of course the buckling capacity about the *y*-*y* axis has already been determined from Example 1, so it remains to find the capacity about the *x*-*x* axis. The relevant section properties are:

Cross-Sectional Area	58.8 cm^2
Yield stress	275 N/mm^2
Modulus of Elasticity	205 kN/mm^2
Radius of gyration about the *x*-*x* axis	8.81 cm
Robertson Constant for the *x*-*x* axis	3.5

Thus:

$$\lambda = \frac{5600}{88.1} = 63.6$$

The limiting slenderness is the same as in Example 1 since *E* nor p_y change:

$$\lambda_0 = 17.48$$

The Perry Factor (equation (38)) is:

$$\eta = \frac{a(\lambda - \lambda_0)}{1000} = \frac{3.5(63.6 - 17.48)}{1000} = 0.16$$

The Euler stress (equation (17) is:

$$p_E = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \cdot 205 \times 10^3}{63.6^2} = 500.8 \text{ N/mm}^2$$

The modifying stress (equation (33)) is:

$$\phi = \frac{p_y + (\eta + 1)\, p_E}{2} = \frac{265 + (0.16 + 1)500.8}{2} = 423.3 \text{ N/mm}^2$$

And so the compressive strength (equations (34)or (40)):

$$p_c = \frac{p_E\, p_y}{\phi + \sqrt{\phi^2 - p_E\, p_y}} = \frac{500.8 \cdot 265}{423.3 + \sqrt{423.3^2 - 500.8 \cdot 265}} = 207.7 \text{ N/mm}^2$$

Thus the load to cause buckling about the *x-x* axis is:

$$P = A_g\, p_c = \frac{5880 \cdot 207.7}{10^3} = 1221.5 \text{ kN}$$

Check this value using Table 24.

Since the buckling capacities are:

- *x-x* axis: $P = 1221.5 \text{ kN}$;
- *y-y* axis $P = 640.4 \text{ kN}$;

the column will first buckle about the *y-y* axis and so its overall buckling capacity is:

$$P = 640.4 \text{ kN}$$

Example 3

Problem

To increase the capacity of the column of Example 2, the supports in the *y*-*y* axis have been changed to fixed-fixed. Determine the buckling capacity.

Solution

As always, sketch the column:

We know the buckling capacity about the *x-x* axis from Example 2, but since the support conditions for the *y-y* axis have changed, the buckling capacity about the *y-y* axis changes. Again the relevant properties are:

Cross-Sectional Area	58.8 cm^2
Yield stress	275 N/mm^2
Modulus of Elasticity	205 kN/mm^2
Radius of gyration about the *y-y* axis	5.12 cm
Robertson Constant for the *y-y* axis	5.5

In this case, since the restraints are fixed-fixed, the effective length is:

$$L_E = 0.7L = 0.7 \cdot 5600 = 3920 \text{ mm}$$

Thus the slenderness is:

$$\lambda = \frac{3920}{51.2} = 76.6$$

The limiting slenderness is the same as before, $\lambda_0 = 17.48$. The Perry Factor (equation (38)) is:

$$\eta = \frac{a\left(\lambda - \lambda_0\right)}{1000} = \frac{5.5\left(76.6 - 17.48\right)}{1000} = 0.32$$

The Euler stress (equation (17) is:

$$p_E = \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 \cdot 205 \times 10^3}{76.6^2} = 345.2 \text{ N/mm}^2$$

The modifying stress (equation (33)) is:

$$\phi = \frac{p_y + (\eta + 1) p_E}{2} = \frac{265 + (0.32 + 1)345.2}{2} = 361.2 \text{ N/mm}^2$$

And so the compressive strength (equations (34)or (40)):

$$p_c = \frac{p_E p_y}{\phi + \sqrt{\phi^2 - p_E p_y}} = \frac{345.2 \cdot 265}{361.2 + \sqrt{361.2^2 - 345.2 \cdot 265}} = 163.8 \text{ N/mm}^2$$

Thus the load to cause buckling about the *y*-*y* axis is thus:

$$P = A_g p_c = \frac{5880 \cdot 163.8}{10^3} = 962.9 \text{ kN}$$

Check this value using Table 24.

Since the buckling capacities are:

- *x*-*x* axis: $P = 1221.5$ kN - still as per Example 2;
- *y*-*y* axis $P = 962.9$ kN - changed due to new support conditions;

thus, the column will first buckle about the *y*-*y* axis and so its overall buckling capacity is:

$$P = 962.9 \text{ kN}$$

Notice that a change in support conditions has resulted in a nearly 50% increase in capacity.

4. Appendix

4.1 Solutions to Differential Equations

The Homogenous Equation

To find the solution of:

$$\frac{d^2y}{dx^2} + k^2 y = 0 \tag{41}$$

we try $y = e^{\lambda x}$ (note that this λ has nothing to do with slenderness but is the conventional mathematical notation for this problem). Thus we have:

$$\frac{dy}{dx} = \lambda e^{\lambda x}; \qquad \frac{d^2y}{dx^2} = \lambda^2 e^{\lambda x}$$

Substituting this into (41) gives:

$$\lambda^2 e^{\lambda x} + k^2 e^{\lambda x} = 0$$

And so we get the characteristic equation by dividing out $e^{\lambda x}$:

$$\lambda^2 + k^2 = 0$$

From which:

$$\lambda = \pm\sqrt{-k^2}$$

Or,

$$\lambda_1 = +ik; \qquad \lambda_2 = -ik$$

Where $i = \sqrt{-1}$. Since these are both solutions, they are both valid and the expression for *y* becomes:

$$y = A_1 e^{ikx} + A_2 e^{-ikx} \tag{42}$$

In which A_1 and A_2 are constants to be determined from the initial conditions of the problem. Introducing Euler's equations:

$$\begin{aligned} e^{ikx} &= \cos kx + i \sin kx \\ e^{-ikx} &= \cos kx - i \sin kx \end{aligned} \tag{43}$$

into (42) gives us:

$$y = A_1 \left(\cos kx + i \sin kx \right) + A_2 \left(\cos kx - i \sin kx \right)$$

Collecting terms:

$$y = \left(A_1 + A_2 \right) \cos kx + \left(iA_1 - iA_2 \right) \sin kx$$

Since the coefficients of the trigonometric functions are constants we can just write:

$$y = A \cos kx + B \sin kx \tag{44}$$

Which is the solution presented in equation (4).

Particular Solution of Non-homogenous Equation

We seek here to show the particular solution to the problem:

$$\frac{d^2y}{dx^2} + k^2 y = A\sin\lambda x \tag{45}$$

Is given by:

$$y = B\sin\lambda x \tag{46}$$

From (46):

$$\frac{dy}{dx} = \lambda B\cos\lambda x; \qquad \frac{d^2y}{dx^2} = -\lambda^2 B\sin\lambda x$$

Substitution into (45) gives:

$$-\lambda^2 B\sin\lambda x + k^2 B\sin\lambda x = A\sin\lambda x \tag{47}$$

Dividing out the common term:

$$-\lambda^2 B + k^2 B = A \tag{48}$$

And so:

$$B = \frac{A}{-\lambda^2 + k^2} \tag{49}$$

Thus (46) becomes:

$$y = \frac{A}{-\lambda^2 + k^2} \sin \lambda x \tag{50}$$

Looking at equation (19), we see that:

$$A \equiv -k^2 a$$
$$\lambda \equiv \frac{\pi}{L}$$

And so the particular solution of equation (19) becomes:

$$y = \frac{-k^2 a}{-\left(\frac{\pi}{L}\right)^2 + k^2} \sin \lambda x \tag{51}$$

Which is given previously as equation (21).

4.2 Code Extracts

Table 23 — Allocation of strut curve

Type of section	Maximum thickness (see note 1)	Axis of buckling	
		x-x	y-y
Hot-finished structural hollow section		a)	a)
Cold-formed structural hollow section		c)	c)
Rolled I-section	≤ 40 mm	a)	b)
	>40 mm	b)	c)
Rolled H-section	≤ 40 mm	b)	c)
	>40 mm	c)	d)
Welded I or H-section (see note 2 and 4.7.5)	≤ 40 mm	b)	c)
	>40 mm	b)	d)
Rolled I-section with welded flange cover plates with $0.25 < U/B < 0.8$ as shown in Figure 14a)	≤ 40 mm	a)	b)
	>40 mm	b)	c)
Rolled H-section with welded flange cover plates with $0.25 < U/B < 0.8$ as shown in Figure 14a)	≤ 40 mm	b)	c)
	>40 mm	c)	d)
Rolled I or H-section with welded flange cover plates with $U/B \geq 0.8$ as shown in Figure 14b)	≤ 40 mm	b)	a)
	>40 mm	c)	b)
Rolled I or H-section with welded flange cover plates with $U/B \leq 0.25$ as shown in Figure 14c)	≤ 40 mm	b)	c)
	>40 mm	b)	d)
Welded box section (see note 3 and 4.7.5)	≤ 40 mm	b)	b)
	>40 mm	c)	c)
Round, square or flat bar	≤ 40 mm	b)	b)
	>40 mm	c)	c)
Rolled angle, channel or T-section Two rolled sections laced, battened or back-to-back Compound rolled sections		Any axis: c)	

NOTE 1 For thicknesses between 40 mm and 50 mm the value of p_c may be taken as the average of the values for thicknesses up to 40 mm and over 40 mm for the relevant value of p_y.

NOTE 2 For welded I or H-sections with their flanges thermally cut by machine without subsequent edge grinding or machining, for buckling about the y-y axis, strut curve b) may be used for flanges up to 40 mm thick and strut curve c) for flanges over 40 mm thick.

NOTE 3 The category "welded box section" includes any box section fabricated from plates or rolled sections, provided that all of the longitudinal welds are near the corners of the cross-section. Box sections with longitudinal stiffeners are NOT included in this category.

Table 22 — Nominal effective length L_E for a compression member[a]

a) non-sway mode			
Restraint (in the plane under consideration) by other parts of the structure			L_E
Effectively held in position at both ends	Effectively restrained in direction at both ends		$0.7L$
	Partially restrained in direction at both ends		$0.85L$
	Restrained in direction at one end		$0.85L$
	Not restrained in direction at either end		$1.0L$
b) sway mode			
One end	Other end		L_E
Effectively held in position and restrained in direction	Not held in position	Effectively restrained in direction	$1.2L$
		Partially restrained in direction	$1.5L$
		Not restrained in direction	$2.0L$

[a] Excluding angle, channel or T-section struts designed in accordance with **4.7.10**.

Annex C (normative)
Compressive strength

C.1 Strut formula

The compressive strength p_c should be taken as the smaller root of:

$$(p_E - p_c)(p_y - p_c) = \eta p_E p_c$$

from which the value of p_c may be obtained using:

$$p_c = \frac{p_E p_y}{\phi + (\phi^2 - p_E p_y)^{0.5}}$$

in which:

$$\phi = \frac{p_y + (\eta + 1)p_E}{2}$$

$$p_E = (\pi^2 E/\lambda^2)$$

where

p_y is the design strength;

λ is the slenderness, see **4.7.2**.

C.2 Perry factor and Robertson constant

The Perry factor η for flexural buckling under axial force should be taken as:

$$\eta = a(\lambda - \lambda_0)/1\,000 \quad \text{but} \quad \eta \geq 0$$

in which the limiting slenderness λ_0 should be taken as $0.2(\pi^2 E/p_y)^{0.5}$.

The Robertson constant a should be taken as follows:

— for strut curve (a): $a = 2.0$;
— for strut curve (b): $a = 3.5$;
— for strut curve (c): $a = 5.5$;
— for strut curve (d): $a = 8.0$.

Table 24 — Compressive strength p_c (N/mm²)

1) Values of p_c in N/mm² with $\lambda < 110$ for strut curve a

λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
15	235	245	255	265	275	315	325	335	345	355	399	409	429	439	458
20	234	244	254	264	273	312	322	332	342	351	395	405	424	434	453
25	232	241	251	261	270	309	318	328	338	347	390	400	419	429	448
30	229	239	248	258	267	305	315	324	333	343	385	395	414	423	442
35	226	236	245	254	264	301	310	320	329	338	380	389	407	416	434
40	223	233	242	251	260	296	305	315	324	333	373	382	399	408	426
42	222	231	240	249	258	294	303	312	321	330	370	378	396	404	422
44	221	230	239	248	257	292	301	310	319	327	366	375	392	400	417
46	219	228	237	246	255	290	299	307	316	325	363	371	388	396	413
48	218	227	236	244	253	288	296	305	313	322	359	367	383	391	407
50	216	225	234	242	251	285	293	302	310	318	355	363	378	386	401
52	215	223	232	241	249	282	291	299	307	315	350	358	373	380	395
54	213	222	230	238	247	279	287	295	303	311	345	353	367	374	388
56	211	220	228	236	244	276	284	292	300	307	340	347	361	368	381
58	210	218	226	234	242	273	281	288	295	303	334	341	354	360	372
60	208	216	224	232	239	269	277	284	291	298	328	334	346	352	364
62	206	214	221	229	236	266	273	280	286	293	321	327	338	344	354
64	204	211	219	226	234	262	268	275	281	288	314	320	330	335	344
66	201	209	216	223	230	257	264	270	276	282	307	312	321	326	334
68	199	206	213	220	227	253	259	265	270	276	299	303	312	316	324
70	196	203	210	217	224	248	254	259	265	270	291	295	303	306	313
72	194	201	207	214	220	243	248	253	258	263	282	286	293	296	302
74	191	198	204	210	216	238	243	247	252	256	274	277	283	286	292
76	188	194	200	206	212	232	237	241	245	249	265	268	274	276	281
78	185	191	197	202	208	227	231	235	239	242	257	259	264	267	271
80	182	188	193	198	203	221	225	229	232	235	248	251	255	257	261
82	179	184	189	194	199	215	219	222	225	228	240	242	246	248	251
84	176	181	185	190	194	209	213	216	219	221	232	234	237	239	242
86	172	177	181	186	190	204	207	209	212	214	224	225	229	230	233
88	169	173	177	181	185	198	200	203	205	208	216	218	220	222	224
90	165	169	173	177	180	192	195	197	199	201	209	210	213	214	216
92	162	166	169	173	176	186	189	191	193	194	201	203	205	206	208
94	158	162	165	168	171	181	183	185	187	188	194	196	198	199	200
96	154	158	161	164	166	175	177	179	181	182	188	189	191	192	193
98	151	154	157	159	162	170	172	173	175	176	181	182	184	185	186
100	147	150	153	155	157	165	167	168	169	171	175	176	178	178	180
102	144	146	149	151	153	160	161	163	164	165	169	170	172	172	174
104	140	142	145	147	149	155	156	158	159	160	164	165	166	166	168
106	136	139	141	143	145	150	152	153	154	155	158	159	160	161	162
108	133	135	137	139	141	146	147	148	149	150	153	154	155	156	157

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

2) Values of p_c (N/mm²) with $\lambda \geq 110$ for strut curve a															
λ	Steel grade and design strength p_y (in N/mm²)														
	S 275					S 355					S 460				
	235	**245**	**255**	**265**	**275**	**315**	**325**	**335**	**345**	**355**	**400**	**410**	**430**	**440**	**460**
110	130	132	133	135	137	142	143	144	144	145	148	149	150	150	151
112	126	128	130	131	133	137	138	139	140	141	144	144	145	146	146
114	123	125	126	128	129	133	134	135	136	136	139	140	141	141	142
116	120	121	123	124	125	129	130	131	132	132	135	135	136	137	137
118	117	118	120	121	122	126	126	127	128	128	131	131	132	132	133
120	114	115	116	118	119	122	123	123	124	125	127	127	128	128	129
122	111	112	113	114	115	119	119	120	120	121	123	123	124	124	125
124	108	109	110	111	112	115	116	116	117	117	119	120	120	121	121
126	105	106	107	108	109	112	113	113	114	114	116	116	117	117	118
128	103	104	105	105	106	109	109	110	110	111	112	113	113	114	114
130	100	101	102	103	103	106	106	107	107	108	109	110	110	110	111
135	94	95	95	96	97	99	99	100	100	101	102	102	103	103	103
140	88	89	90	90	91	93	93	93	94	94	95	95	96	96	96
145	83	84	84	85	85	87	87	87	88	88	89	89	90	90	90
150	78	79	79	80	80	82	82	82	82	83	83	84	84	84	84
155	74	74	75	75	75	77	77	77	77	78	78	79	79	79	79
160	70	70	70	71	71	72	72	73	73	73	74	74	74	74	75
165	66	66	67	67	67	68	68	69	69	69	70	70	70	70	70
170	62	63	63	63	64	64	65	65	65	65	66	66	66	66	66
175	59	59	60	60	60	61	61	61	61	62	62	62	62	63	63
180	56	56	57	57	57	58	58	58	58	58	59	59	59	59	59
185	53	54	54	54	54	55	55	55	55	55	56	56	56	56	56
190	51	51	51	51	52	52	52	52	53	53	53	53	53	53	53
195	48	49	49	49	49	50	50	50	50	50	50	51	51	51	51
200	46	46	46	47	47	47	47	47	48	48	48	48	48	48	48
210	42	42	42	43	43	43	43	43	43	43	44	44	44	44	44
220	39	39	39	39	39	39	39	40	40	40	40	40	40	40	40
230	35	36	36	36	36	36	36	36	36	36	37	37	37	37	37
240	33	33	33	33	33	33	33	33	33	33	34	34	34	34	34
250	30	30	30	30	30	31	31	31	31	31	31	31	31	31	31
260	28	28	28	28	28	28	29	29	29	29	29	29	29	29	29
270	26	26	26	26	26	26	27	27	27	27	27	27	27	27	27
280	24	24	24	24	24	25	25	25	25	25	25	25	25	25	25
290	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23
300	21	21	21	21	21	22	22	22	22	22	22	22	22	22	22
310	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
320	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
330	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
340	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
350	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

3) Values of p_c (N/mm²) with $\lambda < 110$ for strut curve b															
λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	**245**	**255**	**265**	**275**	**315**	**325**	**335**	**345**	**355**	**400**	**410**	**430**	**440**	**460**
15	235	245	255	265	275	315	325	335	345	355	399	409	428	438	457
20	234	243	253	263	272	310	320	330	339	349	391	401	420	429	448
25	229	239	248	258	267	304	314	323	332	342	384	393	411	421	439
30	225	234	243	253	262	298	307	316	325	335	375	384	402	411	429
35	220	229	238	247	256	291	300	309	318	327	366	374	392	400	417
40	216	224	233	241	250	284	293	301	310	318	355	364	380	388	404
42	213	222	231	239	248	281	289	298	306	314	351	359	375	383	399
44	211	220	228	237	245	278	286	294	302	310	346	354	369	377	392
46	209	218	226	234	242	275	283	291	298	306	341	349	364	371	386
48	207	215	223	231	239	271	279	287	294	302	336	343	358	365	379
50	205	213	221	229	237	267	275	283	290	298	330	337	351	358	372
52	203	210	218	226	234	264	271	278	286	293	324	331	344	351	364
54	200	208	215	223	230	260	267	274	281	288	318	325	337	344	356
56	198	205	213	220	227	256	263	269	276	283	312	318	330	336	347
58	195	202	210	217	224	252	258	265	271	278	305	311	322	328	339
60	193	200	207	214	221	247	254	260	266	272	298	304	314	320	330
62	190	197	204	210	217	243	249	255	261	266	291	296	306	311	320
64	187	194	200	207	213	238	244	249	255	261	284	289	298	302	311
66	184	191	197	203	210	233	239	244	249	255	276	281	289	294	301
68	181	188	194	200	206	228	233	239	244	249	269	273	281	285	292
70	178	185	190	196	202	223	228	233	238	242	261	265	272	276	282
72	175	181	187	193	198	218	223	227	232	236	254	257	264	267	273
74	172	178	183	189	194	213	217	222	226	230	246	249	255	258	264
76	169	175	180	185	190	208	212	216	220	223	238	241	247	250	255
78	166	171	176	181	186	203	206	210	214	217	231	234	239	241	246
80	163	168	172	177	181	197	201	204	208	211	224	226	231	233	237
82	160	164	169	173	177	192	196	199	202	205	217	219	223	225	229
84	156	161	165	169	173	187	190	193	196	199	210	212	216	218	221
86	153	157	161	165	169	182	185	188	190	193	203	205	208	210	213
88	150	154	158	161	165	177	180	182	185	187	196	198	201	203	206
90	146	150	154	157	161	172	175	177	179	181	190	192	195	196	199
92	143	147	150	153	156	167	170	172	174	176	184	185	188	189	192
94	140	143	147	150	152	162	165	167	169	171	178	179	182	183	185
96	137	140	143	146	148	158	160	162	164	165	172	173	176	177	179
98	134	137	139	142	145	153	155	157	159	160	167	168	170	171	173
100	130	133	136	138	141	149	151	152	154	155	161	162	164	165	167
102	127	130	132	135	137	145	146	148	149	151	156	157	159	160	162
104	124	127	129	131	133	141	142	144	145	146	151	152	154	155	156
106	121	124	126	128	130	137	138	139	141	142	147	148	149	150	151
108	118	121	123	125	126	133	134	135	137	138	142	143	144	145	147

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

4) Values of p_c (N/mm²) with $\lambda \geq$ 110 for strut curve b															
λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
110	115	118	120	121	123	129	130	131	133	134	138	139	140	141	142
112	113	115	117	118	120	125	127	128	129	130	134	134	136	136	138
114	110	112	114	115	117	122	123	124	125	126	130	130	132	132	133
116	107	109	111	112	114	119	120	121	122	122	126	126	128	128	129
118	105	106	108	109	111	115	116	117	118	119	122	123	124	124	125
120	102	104	105	107	108	112	113	114	115	116	119	119	120	121	122
122	100	101	103	104	105	109	110	111	112	112	115	116	117	117	118
124	97	99	100	101	102	106	107	108	109	109	112	112	113	114	115
126	95	96	98	99	100	103	104	105	106	106	109	109	110	111	111
128	93	94	95	96	97	101	101	102	103	103	106	106	107	107	108
130	90	92	93	94	95	98	99	99	100	101	103	103	104	105	105
135	85	86	87	88	89	92	93	93	94	94	96	97	97	98	98
140	80	81	82	83	84	86	87	87	88	88	90	90	91	91	92
145	76	77	78	78	79	81	82	82	83	83	84	85	85	86	86
150	72	72	73	74	74	76	77	77	78	78	79	80	80	80	81
155	68	69	69	70	70	72	72	73	73	73	75	75	75	76	76
160	64	65	65	66	66	68	68	69	69	69	70	71	71	71	72
165	61	62	62	62	63	64	65	65	65	65	66	67	67	67	68
170	58	58	59	59	60	61	61	61	62	62	63	63	63	64	64
175	55	55	56	56	57	58	58	58	59	59	60	60	60	60	60
180	52	53	53	53	54	55	55	55	56	56	56	57	57	57	57
185	50	50	51	51	51	52	52	53	53	53	54	54	54	54	54
190	48	48	48	48	49	50	50	50	50	50	51	51	51	51	52
195	45	46	46	46	46	47	47	48	48	48	49	49	49	49	49
200	43	44	44	44	44	45	45	45	46	46	46	46	47	47	47
210	40	40	40	40	41	41	41	41	42	42	42	42	42	43	43
220	36	37	37	37	37	38	38	38	38	38	39	39	39	39	39
230	34	34	34	34	34	35	35	35	35	35	35	36	36	36	36
240	31	31	31	31	32	32	32	32	32	32	33	33	33	33	33
250	29	29	29	29	29	30	30	30	30	30	30	30	30	30	30
260	27	27	27	27	27	27	28	28	28	28	28	28	28	28	28
270	25	25	25	25	25	26	26	26	26	26	26	26	26	26	26
280	23	23	23	23	24	24	24	24	24	24	24	24	24	24	24
290	22	22	22	22	22	22	22	22	22	22	23	23	23	23	23
300	20	20	21	21	21	21	21	21	21	21	21	21	21	21	21
310	19	19	19	19	19	20	20	20	20	20	20	20	20	20	20
320	18	18	18	18	18	18	18	19	19	19	19	19	19	19	19
330	17	17	17	17	17	17	17	17	17	18	18	18	18	18	18
340	16	16	16	16	16	16	16	16	17	17	17	17	17	17	17
350	15	15	15	15	15	16	16	16	16	16	16	16	16	16	16

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

5) Values of p_c (N/mm²) with λ < 110 for strut curve c

λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	**245**	**255**	**265**	**275**	**315**	**325**	**335**	**345**	**355**	**400**	**410**	**430**	**440**	**460**
15	235	245	255	265	275	315	325	335	345	355	398	408	427	436	455
20	233	242	252	261	271	308	317	326	336	345	387	396	414	424	442
25	226	235	245	254	263	299	308	317	326	335	375	384	402	410	428
30	220	228	237	246	255	289	298	307	315	324	363	371	388	396	413
35	213	221	230	238	247	280	288	296	305	313	349	357	374	382	397
40	206	214	222	230	238	270	278	285	293	301	335	343	358	365	380
42	203	211	219	227	235	266	273	281	288	296	329	337	351	358	373
44	200	208	216	224	231	261	269	276	284	291	323	330	344	351	365
46	197	205	213	220	228	257	264	271	279	286	317	324	337	344	357
48	195	202	209	217	224	253	260	267	274	280	311	317	330	337	349
50	192	199	206	213	220	248	255	262	268	275	304	310	323	329	341
52	189	196	203	210	217	244	250	257	263	270	297	303	315	321	333
54	186	193	199	206	213	239	245	252	258	264	291	296	308	313	324
56	183	189	196	202	209	234	240	246	252	258	284	289	300	305	315
58	179	186	192	199	205	229	235	241	247	252	277	282	292	297	306
60	176	183	189	195	201	225	230	236	241	247	270	274	284	289	298
62	173	179	185	191	197	220	225	230	236	241	262	267	276	280	289
64	170	176	182	188	193	215	220	225	230	235	255	260	268	272	280
66	167	173	178	184	189	210	215	220	224	229	248	252	260	264	271
68	164	169	175	180	185	205	210	214	219	223	241	245	252	256	262
70	161	166	171	176	181	200	204	209	213	217	234	238	244	248	254
72	157	163	168	172	177	195	199	203	207	211	227	231	237	240	246
74	154	159	164	169	173	190	194	198	202	205	220	223	229	232	238
76	151	156	160	165	169	185	189	193	196	200	214	217	222	225	230
78	148	152	157	161	165	180	184	187	191	194	207	210	215	217	222
80	145	149	153	157	161	176	179	182	185	188	201	203	208	210	215
82	142	146	150	154	157	171	174	177	180	183	195	197	201	203	207
84	139	142	146	150	154	167	169	172	175	178	189	191	195	197	201
86	135	139	143	146	150	162	165	168	170	173	183	185	189	190	194
88	132	136	139	143	146	158	160	163	165	168	177	179	183	184	187
90	129	133	136	139	142	153	156	158	161	163	172	173	177	178	181
92	126	130	133	136	139	149	152	154	156	158	166	168	171	173	175
94	124	127	130	133	135	145	147	149	151	153	161	163	166	167	170
96	121	124	127	129	132	141	143	145	147	149	156	158	160	162	164
98	118	121	123	126	129	137	139	141	143	145	151	153	155	157	159
100	115	118	120	123	125	134	135	137	139	140	147	148	151	152	154
102	113	115	118	120	122	130	132	133	135	136	143	144	146	147	149
104	110	112	115	117	119	126	128	130	131	133	138	139	142	142	144
106	107	110	112	114	116	123	125	126	127	129	134	135	137	138	140
108	105	107	109	111	113	120	121	123	124	125	130	131	133	134	136

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

6) Values of p_c (N/mm²) with $\lambda \geq 110$ for strut curve c															
λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	245	255	265	275	315	325	335	345	355	400	410	430	440	460
110	102	104	106	108	110	116	118	119	120	122	126	127	129	130	132
112	100	102	104	106	107	113	115	116	117	118	123	124	125	126	128
114	98	100	101	103	105	110	112	113	114	115	119	120	122	123	124
116	95	97	99	101	102	108	109	110	111	112	116	117	118	119	120
118	93	95	97	98	100	105	106	107	108	109	113	114	115	116	117
120	91	93	94	96	97	102	103	104	105	106	110	110	112	112	113
122	89	90	92	93	95	99	100	101	102	103	107	107	109	109	110
124	87	88	90	91	92	97	98	99	100	100	104	104	106	106	107
126	85	86	88	89	90	94	95	96	97	98	101	102	103	103	104
128	83	84	86	87	88	92	93	94	95	95	98	99	100	100	101
130	81	82	84	85	86	90	91	91	92	93	96	96	97	98	99
135	77	78	79	80	81	84	85	86	87	87	90	90	91	92	92
140	72	74	75	76	76	79	80	81	81	82	84	85	85	86	87
145	69	70	71	71	72	75	76	76	77	77	79	80	80	81	81
150	65	66	67	68	68	71	71	72	72	73	75	75	76	76	76
155	62	63	63	64	65	67	67	68	68	69	70	71	71	72	72
160	59	59	60	61	61	63	64	64	65	65	66	67	67	67	68
165	56	56	57	58	58	60	60	61	61	61	63	63	64	64	64
170	53	54	54	55	55	57	57	58	58	58	60	60	60	60	61
175	51	51	52	52	53	54	54	55	55	55	56	57	57	57	58
180	48	49	49	50	50	51	52	52	52	53	54	54	54	54	55
185	46	46	47	47	48	49	49	50	50	50	51	51	52	52	52
190	44	44	45	45	45	47	47	47	47	48	49	49	49	49	49
195	42	42	43	43	43	45	45	45	45	45	46	46	47	47	47
200	40	41	41	41	42	43	43	43	43	43	44	44	45	45	45
210	37	37	38	38	38	39	39	39	40	40	40	40	41	41	41
220	34	34	35	35	35	36	36	36	36	36	37	37	37	37	38
230	31	32	32	32	32	33	33	33	33	34	34	34	34	34	35
240	29	29	30	30	30	30	31	31	31	31	31	31	32	32	32
250	27	27	27	28	28	28	28	28	29	29	29	29	29	29	29
260	25	25	26	26	26	26	26	26	27	27	27	27	27	27	27
270	23	24	24	24	24	24	25	25	25	25	25	25	25	25	25
280	22	22	22	22	22	23	23	23	23	23	23	24	24	24	24
290	21	21	21	21	21	21	21	22	22	22	22	22	22	22	22
300	19	19	20	20	20	20	20	20	20	20	21	21	21	21	21
310	18	18	18	19	19	19	19	19	19	19	19	19	19	19	20
320	17	17	17	17	18	18	18	18	18	18	18	18	18	18	18
330	16	16	16	16	17	17	17	17	17	17	17	17	17	17	17
340	15	15	15	16	16	16	16	16	16	16	16	16	16	16	16
350	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

7) Values of p_c (N/mm²) with $\lambda < 110$ for strut curve d															
λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	**245**	**255**	**265**	**275**	**315**	**325**	**335**	**345**	**355**	**400**	**410**	**430**	**440**	**460**
15	235	245	255	265	275	315	325	335	345	355	397	407	425	435	453
20	232	241	250	259	269	305	314	323	332	341	381	390	408	417	434
25	223	231	240	249	257	292	301	309	318	326	365	373	390	398	415
30	213	222	230	238	247	279	287	296	304	312	348	356	372	380	396
35	204	212	220	228	236	267	274	282	290	297	331	339	353	361	375
40	195	203	210	218	225	254	261	268	275	283	314	321	334	341	355
42	192	199	206	214	221	249	256	263	270	277	307	314	327	333	346
44	188	195	202	209	216	244	251	257	264	271	300	306	319	325	337
46	185	192	199	205	212	239	245	252	258	265	293	299	311	317	329
48	181	188	195	201	208	234	240	246	252	259	286	291	303	309	320
50	178	184	191	197	204	228	235	241	247	253	278	284	295	301	311
52	174	181	187	193	199	223	229	235	241	246	271	277	287	292	303
54	171	177	183	189	195	218	224	229	235	240	264	269	279	284	294
56	167	173	179	185	191	213	219	224	229	234	257	262	271	276	285
58	164	170	175	181	187	208	213	218	224	229	250	255	264	268	277
60	161	166	172	177	182	203	208	213	218	223	243	247	256	260	268
62	157	163	168	173	178	198	203	208	212	217	236	240	248	252	260
64	154	159	164	169	174	193	198	202	207	211	229	233	241	245	252
66	150	156	160	165	170	188	193	197	201	205	223	226	234	237	244
68	147	152	157	162	166	184	188	192	196	200	216	220	226	230	236
70	144	149	153	158	162	179	183	187	190	194	210	213	219	222	228
72	141	145	150	154	158	174	178	182	185	189	203	207	213	215	221
74	138	142	146	150	154	170	173	177	180	183	197	200	206	209	214
76	135	139	143	147	151	165	169	172	175	178	191	194	199	202	207
78	132	136	139	143	147	161	164	167	170	173	186	188	193	195	200
80	129	132	136	140	143	156	160	163	165	168	180	182	187	189	194
82	126	129	133	136	140	152	155	158	161	163	175	177	181	183	187
84	123	126	130	133	136	148	151	154	156	159	169	171	176	177	181
86	120	123	127	130	133	144	147	149	152	154	164	166	170	172	175
88	117	120	123	127	129	140	143	145	148	150	159	161	165	167	170
90	114	118	121	123	126	137	139	141	144	146	154	156	160	161	164
92	112	115	118	120	123	133	135	137	139	142	150	152	155	156	159
94	109	112	115	117	120	129	132	134	136	138	145	147	150	152	154
96	107	109	112	115	117	126	128	130	132	134	141	143	146	147	150
98	104	107	109	112	114	123	125	126	128	130	137	138	141	143	145
100	102	104	107	109	111	119	121	123	125	126	133	134	137	138	141
102	99	102	104	106	108	116	118	120	121	123	129	131	133	134	136
104	97	99	102	104	106	113	115	116	118	120	126	127	129	130	132
106	95	97	99	101	103	110	112	113	115	116	122	123	125	126	128
108	93	95	97	99	101	107	109	110	112	113	119	120	122	123	125

Table 24 — Compressive strength p_c (N/mm²) (*continued*)

8) Values of p_c (N/mm²) with $\lambda \geq 110$ for strut curve d

λ	Steel grade and design strength p_y (N/mm²)														
	S 275					S 355					S 460				
	235	**245**	**255**	**265**	**275**	**315**	**325**	**335**	**345**	**355**	**400**	**410**	**430**	**440**	**460**
110	91	93	95	96	98	105	106	108	109	110	115	116	118	119	121
112	88	90	92	94	96	102	103	105	106	107	112	113	115	116	118
114	86	88	90	92	94	99	101	102	103	104	109	110	112	113	114
116	85	86	88	90	91	97	98	99	101	102	106	107	109	110	111
118	83	84	86	88	89	95	96	97	98	99	103	104	106	107	108
120	81	82	84	86	87	92	93	94	95	96	101	101	103	104	105
122	79	81	82	84	85	90	91	92	93	94	98	99	100	101	102
124	77	79	80	82	83	88	89	90	91	92	95	96	98	98	99
126	76	77	78	80	81	86	87	88	89	89	93	94	95	96	97
128	74	75	77	78	79	84	85	85	86	87	91	91	93	93	94
130	72	74	75	76	77	82	83	83	84	85	88	89	90	91	92
135	68	70	71	72	73	77	78	79	79	80	83	84	85	85	86
140	65	66	67	68	69	73	73	74	75	75	78	79	80	80	81
145	62	63	64	65	65	69	69	70	71	71	74	74	75	75	76
150	59	60	60	61	62	65	66	66	67	67	69	70	71	71	72
155	56	57	57	58	59	62	62	63	63	64	66	66	67	67	68
160	53	54	55	55	56	58	59	59	60	60	62	62	63	63	64
165	50	51	52	53	53	55	56	56	57	57	59	59	60	60	61
170	48	49	49	50	51	53	53	54	54	54	56	56	57	57	57
175	46	47	47	48	48	50	51	51	51	52	53	53	54	54	55
180	44	45	45	46	46	48	48	49	49	49	50	51	51	51	52
185	42	43	43	44	44	46	46	46	47	47	48	48	49	49	49
190	40	41	41	42	42	44	44	44	44	45	46	46	46	47	47
195	38	39	39	40	40	42	42	42	42	43	44	44	44	45	45
200	37	37	38	38	39	40	40	40	41	41	42	42	42	43	43
210	34	34	35	35	35	37	37	37	37	37	38	38	39	39	39
220	31	32	32	32	33	34	34	34	34	34	35	35	36	36	36
230	29	29	30	30	30	31	31	31	32	32	32	33	33	33	33
240	27	27	28	28	28	29	29	29	29	29	30	30	30	30	31
250	25	25	26	26	26	27	27	27	27	27	28	28	28	28	28
260	24	24	24	24	24	25	25	25	25	25	26	26	26	26	26
270	22	22	22	23	23	23	23	23	24	24	24	24	24	24	25
280	21	21	21	21	21	22	22	22	22	22	23	23	23	23	23
290	19	20	20	20	20	20	21	21	21	21	21	21	21	21	21
300	18	18	19	19	19	19	19	19	19	20	20	20	20	20	20
310	17	17	17	18	18	18	18	18	18	18	19	19	19	19	19
320	16	16	16	17	17	17	17	17	17	17	18	18	18	18	18
330	15	15	16	16	16	16	16	16	16	16	17	17	17	17	17
340	15	15	15	15	15	15	15	15	15	15	16	16	16	16	16
350	14	14	14	14	14	14	14	15	15	15	15	15	15	15	15

4.3 Past Exam Questions

Semester 1 Paper 2007/8

(a) Briefly explain how to calculate the second moment of area of a section comprised of simple shapes. (3 marks)
(b) Describe what is meant by complimentary shear stresses. (4 marks)
(c) Determine the maximum load *w* that can be sustained by the column *BD* shown in Fig. Q.4. (18 marks)
The relevant section properties are given in Table Q4 and the Perry-Robertson formula is given.

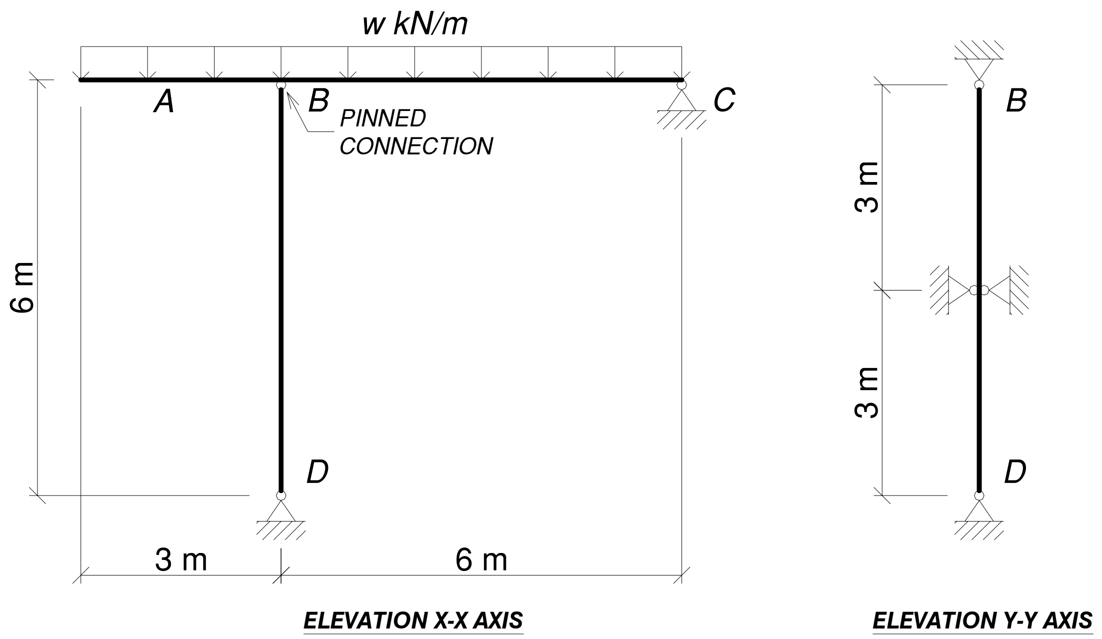


FIG. Q4

Table Q4

Relevant Section Properties for 152 × 152 × 23 UC			
Cross-Sectional Area	29.2 cm^2	Radius of gyration about the *x-x* axis	6.54 cm
Yield stress	275 N/mm^2	Radius of gyration about the *y-y* axis	3.70 cm
Modulus of Elasticity	205 kN/mm^2	Robertson Constant for the *x-x* axis	5.5
		Robertson Constant for the *y-y* axis	5.5

Perry-Robertson Strut Formula:

$$p_c = \frac{p_E p_y}{\phi + \sqrt{\phi^2 - p_E p_y}}$$

where $\phi = \dfrac{p_y + (\eta + 1) p_E}{2}$; $\quad p_E = \dfrac{\pi^2 E}{\lambda^2}$; $\quad \eta = \dfrac{a(\lambda - \lambda_0)}{1000}$; $\quad \lambda_0 = 0.2\sqrt{\dfrac{\pi^2 E}{p_y}}$

In which the symbols have their usual meanings.